A Theory of Capital Controls as
Dynamic Terms-of-Trade Manipulation[*]

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Abstract

We develop a theory of capital controls as dynamic terms-of-trade manipulation. We study an infinite horizon endowment economy with two countries. One country chooses taxes on international capital flows in order to maximize the welfare of its representative agent, while the other country is passive. In this neoclassical benchmark model we show that capital controls are not guided by the absolute desire to alter the intertemporal price of the goods produced in any given period, but rather by the relative strength of this desire between two consecutive periods. Specifically, a country growing faster than the rest of the world has incentives to promote domestic savings by taxing capital inflows or subsidizing capital outflows. Although our theory of capital controls emphasizes interest rate manipulation, the pattern of borrowing and lending, per se, is irrelevant.

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1 Introduction

Since the end of World War II, bilateral and multilateral trade agreements have led to dramatic tariff reductions around the world, contributing to a spectacular increase in world trade; see Subramanian and Wei (2007) and Baier and Bergstrand (2007). Starting in the mid-1980s, the world has also experienced a dramatic increase in capital markets integration, with increased cross-border flows both across industrial countries and between industrial and developing countries; see Kose, Prasad, Rogoff, and Wei (2009). In sum, the world has experienced a dramatic increase in intratemporal and intertemporal trade, as Figure 1 illustrates.

The multilateral institutions that promote both types of trade, however, have followed two very different approaches. The primary goal of the World Trade Organization (WTO), and its predecessor the General Agreements on Tariffs and Trade, has been to reduce relative price distortions in intratemporal trade. The focus on relative price distortions and their associated terms-of-trade implications in static environments has a long and distinguished history in the international trade literature, going back to Torrens (1844) and Mill (1844). This rich history is echoed by recent theoretical and empirical work emphasizing the role of terms-of-trade manipulation in the analysis of optimal tariffs and its implication for the WTO; see Bagwell and Staiger (1999, 2011) and Broda, Limao, and Weinstein (2008).

By contrast, international efforts toward increased capital openness have emphasized the effects of capital controls on macroeconomic and financial stability. Consequently, the multilateral institutions that promote capital market integration, like the International Monetary Fund (IMF), have taken a different, more nuanced approach to intertemporal trade, as exemplified in the recent IMF recommendations on the appropriate use of capital controls; see Ostry, Ghosh, Habermeier, Chamon, Qureshi, and Reinhardt (2010). Although the terms-of-trade effects emphasized in the international trade literature have natural implications for the analysis of optimal capital controls, these effects play little role in the existing international macro literature.

The objective of this paper is to bridge the gap between the trade approach to tariffs and the macroeconomic approach to capital controls. We do so by developing a neoclassical benchmark model in which the only rationale for capital controls is dynamic terms-of-trade manipulation. Our objective is not to argue that the only motive for observed capital controls is the distortion of relative prices or that the removal of such distortions should be the only goal of international policy coordination. Rather, we want to develop some basic tools to think about capital controls as a form of intertemporal trade policy and
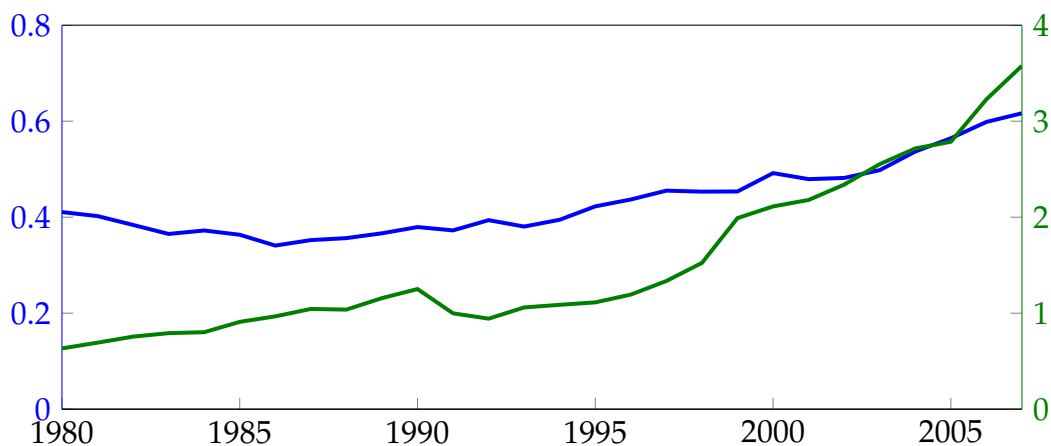
Figure 1: International Trade and Financial Integration

Note: The top (blue) line with the axis on the left represents the sum of world export and imports over world GDP (source: IMF World Economic Outlook). The bottom (green) line with the axis on the right represents the sum of world assets and world liabilities over world GDP (source: updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007))

explore the implications of this idea for how unilaterally optimal capital controls should covary with other macroeconomic variables over time.

The starting point of our paper is that in an Arrow-Debreu economy there is no difference between intertemporal trade and intratemporal trade. In such an environment, one only needs to relabel goods by time period and the same approach used to study static terms-of-trade manipulation can be used to analyze dynamic terms-of-trade manipulation. Our analysis builds on this simple observation together with the time-separable structure of preferences typically used in macro applications.

One key insight that emerges from our analysis is that for a country trading intertemporally, unilaterally optimal capital controls are not guided by the absolute desire to alter the intertemporal price of goods produced in a given period, but rather by the relative strength of this desire between two consecutive periods. If a country is a net seller of goods dated t and $t + 1$ in equal amounts, and faces equal elasticities in both periods, there is no incentive for the country to distort the saving decisions of its consumers at date t. It is the time variation in the incentive to distort intertemporal prices that leads to non-zero capital controls. This is a general principle that, to our knowledge, is novel both to the international macro and international trade literature.

To illustrate this general principle in the simplest possible way, we first consider an infinite horizon, two-country, one-good endowment economy. In this model the only relative prices are real interest rates. We solve for the unilaterally optimal taxes on interna-

tional capital flows in one country, Home, under the assumption that the other country, Foreign, is passive.[1] In this environment, the principle described above has sharp implications for the direction of optimal capital flow taxes. In particular, it is optimal for Home to tax capital inflows (or subsidize capital outflows) in periods in which Home is growing faster than the rest of the world and to tax capital outflows (or subsidize capital inflows) in periods in which it is growing more slowly. Accordingly, if relative endowments converge to a steady state, then taxes on international capital flows converge to zero. Although our theory of capital controls emphasizes interest rate manipulation, the sign of taxes on capital flows only depends on the growth rate of the economy relative to the rest of the world. Home may be a net saver or a net borrower; Home may have a positive or a negative net financial position; if Home grows faster than the rest of the world, it has incentives to promote domestic savings by taxing capital inflows or subsidizing capital outflows.

The intuition for our results is as follows. Consider Home's incentives to distort domestic consumption in each period. In periods of larger trade deficits, it has a stronger incentive, as a buyer, to distort prices downward by lowering domestic consumption. Similarly, in periods of larger trade surpluses, it has a stronger incentive, as a seller, to distort prices upward by raising domestic consumption. Since periods of faster growth at home tend to be associated with either lower future trade deficits or larger future trade surpluses, Home always has an incentive to raise future consumption relative to current consumption in such periods. This is exactly what taxes on capital inflows or subsidies on capital outflows accomplish through their effects on relative distortions across periods.

The second part of our paper explores further the frontier between international macro and international trade policy by introducing multiple goods, thereby allowing for both intertemporal and intratemporal trade. In order to maintain the focus of our analysis on capital controls, we assume that Home can still choose its taxes on capital flows unilaterally, but that it is constrained by a free-trade agreement that prohibits good-specific taxes/subsidies in all periods. In this environment, we show that the incentive to distort trade over time does not depend only on the overall growth of the country's output relative to the world, but also on its composition.

We illustrate the role of these compositional effects in two ways. First, we establish a

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[1]Throughout our analysis, we assume that the home government can freely commit at date 0 to a sequence of taxes. In the economic environment considered in this paper, this is a fairly mild assumption. As we formally establish in Section 3.4, if the home government can enter debt commitments at all maturities, as in Lucas and Stokey (1983), the optimal sequence of taxes under commitment is time-consistent. To the extent that bonds of different maturities are available in practice—and they are—we therefore view the model with commitment as the most natural benchmark for the question that we are interested in.

general formula that relates intertemporal distortions to the covariance between the price elasticities of different goods and the change in the value of home endowments. Ceteris paribus, we show that Home is more likely to raise aggregate consumption if a change in the value of home endowments is tilted towards goods whose prices are more manipulable. In this richer environment, even a country that is too small to affect world interest rate may find it optimal to impose capital controls for terms-of-trade considerations as long as it is large enough to affect some intratemporal prices. Second, we illustrate through a simple analytical example how such compositional issues relate to cross-country differences in demand. In a multi good world in which countries have different preferences, a change in the time profile of consumption not only affects the interest rate but also the relative prices of consumption goods in each given period. This is an effect familiar from the literature on the transfer problem, which goes back to the debate between Keynes (1929) and Ohlin (1929). In our context this means that by distorting its consumers' decision to allocate spending between different periods a country also affects its static terms of trade. Even if all static trade distortions are banned by a free-trade agreement, our analysis demonstrates that, away from steady state, intratemporal prices may not be at their undistorted levels if capital controls are allowed.

We conclude by returning to the issue of capital controls and international cooperation, or lack thereof, alluded to at the beginning of our introduction. We consider the case of capital control wars in which the two countries simultaneously set taxes on capital flows optimally at date 0, taking as given the sequence of taxes chosen by the other country. Using a simple quantitative example, we show that far from canceling each other out, capital controls imposed by both countries aggravate the misallocation of international capital flows.

Our paper attacks an international macroeconomic question following a classical approach from the international trade literature and using tools from the dynamic public finance literature. In international macro, there is a growing theoretical literature demonstrating, among other things, how restrictions on international capital flows may be welfare-enhancing in the presence of various credit market imperfections; see e.g. Calvo and Mendoza (2000), Caballero and Lorenzoni (2007), Aoki, Benigno, and Kiyotaki (2010), Jeanne and Korinek (2010), and Martin and Taddei (2010). In addition to these second-best arguments, there also exists an older literature emphasizing the so-called "trilemma": one cannot have a fixed exchange rate, an independent monetary policy and free capital mobility; see e.g. McKinnon and Oates (1966), or more recently, Obstfeld, Shambaugh, and Taylor (2010). To the extent that having fixed exchange and an independent monetary policy may be welfare-enhancing, such papers offer a distinct rationale for

capital controls.

In related work, Obstfeld and Rogoff (1996) apply optimal tarriff arguments to study capital controls in a two-period, two-country, one-good endowment economy model. In this environment, the optimal tariff argument in trade theory has obvious implications: if a country borrows, it should tax capital inflows to decrease the world interest rate; conversely, if it saves, it should tax outflows to raise it. Our analysis highlights that this basic insight is misleading: it is specific to the two-period model and does not carry over to more general settings. As discussed earlier, it is not a country's status as a borrower or lender—neither in terms of stocks nor flows—that determines the sign of the optimal capital tax. Instead, what matters is the contemporaneous growth rate of a country compared to its trading partner. For instance, a country in steady state may find itself being a debtor or creditor, from past saving choices, yet the optimal capital tax at the steady state is zero. Similarly, a country expecting to catch up in the long run may run a current account deficit today, yet subsidize capital inflows because it expects poor growth in the short run.

On the international trade side, the literature on optimal taxes in open economies is large and varied. The common starting point of most trade policy papers, however, is that international trade is balanced. They therefore abstract from intertemporal considerations.[2] While one could, in principle, go from intratemporal to intertemporal trade policy by relabeling goods in an abstract Arrow-Debreu economy, existing trade policy papers typically focus on low-dimensional general equilibrium models, i.e., with only two goods. Jones (1967) uses optimal tariff arguments to study the taxation of capital movements in a static model with two (final) goods. In his model, international capital flows correspond to imports and exports of physical capital, which can be thought of as a third (intermediate) good. Compared to the present analysis, there is no intertemporal borrowing and lending. Other exceptions featuring more than two goods only offer: (*i*) partial equilibrium results under the assumption of quasi-linear preferences; (*ii*) sufficient conditions under which seemingly paradoxical results may arise, see e.g. Feenstra (1986) and Itoh and Kiyono (1987); or (*iii*) fairly weak restriction on the structure of optimal trade policy, see e.g. Dixit (1985) and Bond (1990). To summarize there are no 'off-the-shelf' results from the existing trade literature that directly apply to the dynamic environment considered in our paper.

In terms of methodology, we follow the dynamic public finance literature and use the primal approach to characterize first optimal wedges rather than explicit policy instru-

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[2]A notable exception is Bagwell and Staiger (1990), though their focus is on self-enforcing trade agreements. See Staiger (1995) for an overview of that literature.

ments; see e.g. Lucas and Stokey (1983). Since there are typically many ways to implement the optimal allocation in an intertemporal context, this approach will help us clarify the equivalence between capital controls and other policy instruments. Our focus on the optimal structure of taxes in an open economy is also related to Anderson (1991) and Anderson and Young (1992). Compared to the present paper, both papers focus on the case of a small-open economy in which the rationale for taxes is the financing of an exogenous stream of government expenditures rather than the manipulation of intertemporal and intratemporal terms-of-trade. Finally, since our theory of capital controls models one of the two governments as a dynamic monopolist optimally choosing the pattern of consumption over time, our analysis bears some resemblance to the problem of a dynamic monopolist optimally choosing the rate of extraction of some exhaustible resources; see Stiglitz (1976).

The rest of our paper is organized as follows. Section 2 describes a simple one-good economy. Section 3 characterizes the structure of optimal capital controls in this environment. Section 4 extends our results to the case of arbitrarily many goods. Section 5 considers the case of capital control wars. Section 6 offers some concluding remarks.

2 Basic Environment

2.1 A Dynamic Endowment Economy

There are two countries, Home and Foreign. Time is discrete and infinite, $t = 0, 1, ...$ and there is no uncertainty. The preferences of the representative consumer at home are represented by the additively separable utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t denotes consumption; u is a twice continuously differentiable, strictly increasing, and strictly concave function, with $\lim_{c \to 0} u'(c) = \infty$; and $\beta \in (0, 1)$ is the discount factor. The preferences of the representative consumer abroad have a similar form, with asterisks denoting foreign variables.

Both domestic and foreign consumers receive an endowment sequence denoted by $\{y_t\}$ and $\{y_t^*\}$, respectively. Endowments are bounded away from zero in all periods in both countries. We make two simplifying assumptions: world endowments are fixed across periods, $y_t + y_t^* = Y$, and the home and foreign consumer have the same discount factor, $\beta = \beta^*$. Accordingly, in the absence of distortions, there should be perfect con-

sumption smoothing across time in both countries.[3]

We assume that both countries begin with zero assets at date 0.[4] Let p_t be the price of a unit of consumption in period t on the world capital markets. In the absence of taxes, the intertemporal budget constraint of the home consumer is

$$\sum_{t=0}^{\infty} p_t(c_t - y_t) \leq 0.$$

The budget constraint of of the foreign consumer is the same expression with asterisks on c_t and y_t.

2.2 A Dynamic Monopolist

For most of the paper, we will focus on the case in which the home government sets taxes on capital flows in order to maximize domestic welfare, assuming the foreign government is passive: it does not have any tax policy in place and does not respond to variations in the home policy. We will look at the case where both governments set taxes strategically in Section 5.

In order to characterize the optimal policy of the home government, we follow the dynamic public finance literature and use the primal approach. That is, we approach the optimal policy problem of the home government by studying a planning problem in which equilibrium quantities are chosen directly and address implementation issues later.

Formally, we assume that the objective of the home government is to maximize the lifetime utility of the representative domestic consumer subject to (i) utility maximization by the foreign consumer at (undistorted) world prices p_t, and (ii) market clearing in each period. The foreign consumer first-order conditions are given by

$$\beta^t u^{*\prime}(c_t^*) = \lambda^* p_t, \tag{1}$$

$$\sum_{t=0}^{\infty} p_t(c_t^* - y_t^*) = 0, \tag{2}$$

where λ^* is the Lagrange multiplier on the foreign consumer's budget constraint. Moreover, goods market clearing requires

$$c_t + c_t^* = Y. \tag{3}$$

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[3]In Section 3.3 we demonstrate that our results generalize to environment with aggregate fluctuations if consumers have Constant Relative Risk Aversion (CRRA) utility.

[4]The assumption of zero initial assets is relaxed in Section 3.4.

Combining equations (1)-(3), we can express the planning problem of the home government as

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad\qquad (P)$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)(c_t - y_t) = 0. \qquad\qquad (4)$$

Equation (4) is an implementability constraint, familiar from the optimal taxation literature. Given a sequence of domestic consumption, condition (4) is sufficient to ensure the existence of a feasible, utility-maximizing consumption sequence for Foreign. The argument is constructive: given $\{c_t\}$, the proposed sequence $\{c_t^*\}$ is obtained from market clearing (3) and the sequence of prices is computed from (1), so that (2) is implied by (4), ensuring that the foreign consumer's sufficient conditions for optimality are met.

The Lagrangian associated with the previous planning problem is given by

$$L = \sum_{t=0}^{\infty} \beta^t u(c_t) + \mu \sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)(y_t - c_t). \qquad\qquad (5)$$

In the next section we will solve (P) by looking for a consumption sequence $\{c_t\}$ that maximizes L. Given the additive separability of preferences, this is equivalent to maximize $u(c_t) + \mu u^{*\prime}(Y - c_t)(y_t - c_t)$ period by period.

In our online Appendix we show that if time is continuous, then any solution of (P) must be a maximand of L. This is sufficient to establish that our key result, Proposition 1, holds under the assumptions of Section 2.1 without further qualification. In discrete time, a similar result can be established if one allows the home government to choose lotteries. In the absence of lotteries, however, the set of maximands of L may not coincide with the set of solutions of (P). To avoid dealing either with lotteries or the technicalities associated with continuous time, we simply assume in the rest of this paper that $u^{*\prime}(Y - c_t)(c_t - y_t)$ is a strictly convex function of c_t for all y_t. This implies that L is strictly concave and that any solution of (P) must be a maximand of L.[5]

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[5]Since any solution of (P) must also be a solution of the relaxed planning problem in which (4) holds as a weak inequality, the previous observation derives from Theorem 1, page 217 in Luenberger (1969).

3 Optimal Capital Controls

3.1 Optimal Allocation

We first describe how home consumption $\{c_t\}$ fluctuates with home endowments $\{y_t\}$ along the optimal path. Next we will show how the optimal allocation can be implemented using taxes on international capital flows.

Under the assumptions that marginal utilities are infinite at zero and that foreign endowments are bounded away from zero, optimal consumption choices must lie in $(0, Y)$ in all periods. Accordingly, we can express the first-order condition associated with the maximization of L as

$$u'(c_t) = \mu \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right], \tag{6}$$

where $\mu > 0$ is the Lagrange multiplier on the implementability constraint. Since u is strictly concave, the left-hand side is strictly decreasing in c_t and since $u^{*\prime}(Y - c_t)(c_t - y_t)$ is strictly convex, the right-hand side is strictly increasing in c_t. Thus, conditional on μ, there exists at most one value of c_t such that equation (6) is satisfied. Since the previous first-order condition must be satisfied by any solution of Home's planning problem, such a solution must be unique as well.[6]

Equation (6) leads to our first observation. Although the entire sequence $\{y_t\}$ affects the *level* of current consumption through their effects on the Lagrange multiplier μ, we see that *changes* in current consumption c_t along the optimal path only depend on *changes* in the current value of y_t.

The next proposition further shows that there is a monotonic relationship between domestic consumption and domestic endowments along the optimal path.

Proposition 1 *(Procyclical consumption) For any two periods t and s, if the home endowment is larger in s, $y_s > y_t$, then the home consumption is also higher, $c_s > c_t$.*

Figure 2 provides a graphical representation of the first order condition associated with Home's planning problem. On the x-axis we have domestic consumption c, which determines foreign consumption, $Y - c$, by market clearing. The downward sloping (blue) curve represents the marginal cost associated with reducing consumption at home by one unit, the left hand side in equation (6). The solid upward sloping (green) curves

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[6]Under the assumption that marginal utilities are infinite at zero and that foreign endowments are bounded away from zero, one can also check that such a solution exists. The formal argument can be found in our online Appendix.
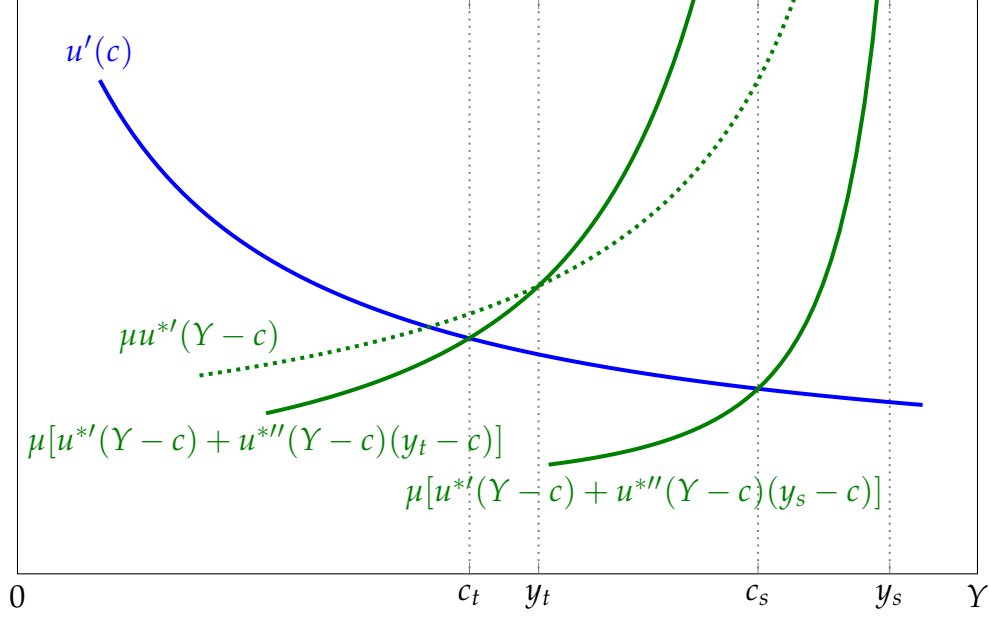
Figure 2: Consumption is Procyclical Along the Optimal Path

represent the marginal benefit associated with reducing consumption at home by one unit, the right hand side in equation (6). This captures both the price of that marginal unit, $u^{*\prime}(Y - c_t)$, as well as the change in the price of the infra marginal units, $u^{*\prime\prime}(Y - c_t)$. The optimal consumption choices at date t and s correspond to the point where the marginal benefit of reducing domestic consumption is equal to its marginal cost. For reference, we also plot $\mu u^{*\prime}(Y - c)$ represented by the upward sloping dotted (green) curve. Its intersection with the downward sloping (blue) curve represents an efficient level of consumption. Its intersection with the upward sloping (green) curve occurs at the point where net sales are zero: $c = y$.

Figure 2 gives the intuition for Proposition 1. As the endowment increases from y_t to y_s, the curve $u^{\prime}(c)$ does not move. At the same time, the marginal benefit curve shifts down, as the price decrease associated to a reduction in c applies to a larger amount of inframarginal units sold. This induces Home to consume more, explaining why consumption is procyclical along the optimal path.[7]

As a preliminary step in the analysis of optimal capital flow taxes, we conclude this

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[7]To see why the strict convexity of $u^{*\prime}(Y - c_t)(c_t - y_t)$ in c_t is not crucial for establishing Proposition 1, note that any sequence $\{c_t\}$ that maximizes L must be such that $c_t \in \arg\max u(c) + \mu u^{*\prime}(Y - c)(y_t - c)$ period by period. Under the assumption that u^* is strictly concave, $u(c) + \mu u^{*\prime}(Y - c)(y_t - c)$ satisfies the single-crossing property in (c, y_t). Thus the set of consumption levels that maximize L must be increasing in y_t in the strong set order. To establish that consumption is procyclical along the optimal path, the only technical question then is whether any solution of (P) can be recovered as a maximand of L for some value of μ. As we already discussed at the end of Section 2.2, the answer in continuous time is always yes.

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section by describing how the "wedge" between the marginal utility of domestic and foreign consumption varies along the optimal path. Formally, define

$$\tau_t \equiv \frac{u'(c_t)}{\mu u^{*\prime}(c_t^*)} - 1. \tag{7}$$

By market clearing, we know that $c_t^* = Y - c_t$. Thus combining the definition of τ_t with the strict concavity of u and u^*, we obtain the following corollary to Proposition 1.

Corollary 1 *(Countercyclical wedges) For any two periods t and s, if the home endowment is larger in s, $y_s > y_t$, then the wedge is lower, $\tau_s < \tau_t$.*

At this point, it is worth pausing to discuss how Corollary 1 relates to and differs from existing results in the trade policy literature. By equations (6) and (7), we have

$$\tau_t = -\frac{u^{*\prime\prime}(Y - c_t)}{u^{*\prime}(Y - c_t)}(c_t - y_t). \tag{8}$$

Condition (8) is closely related to the well-known optimal tariff formula involving the elasticity of the foreign export supply curve in static trade models with two goods and/or quasi-linear preferences. This should not be too surprising since τ_t measures the difference between the marginal utility of domestic and foreign consumption. According to equation (8), the wedge τ_t is positive in periods of trade deficit and negative in periods of trade surplus. This captures the idea that if (time-varying) trade taxes were available, Home would like to tax imports if $c_t - y_t > 0$ and tax exports if $c_t - y_t < 0$. Corollary 1, however, goes beyond this simple observation by establishing a monotonic relationship between τ_t and y_t. This novel insight will play a key role in our analysis of optimal capital controls.

3.2 Optimal Taxes on International Capital Flows

It is well-known from the Ramsey taxation literature that there are typically many combinations of taxes that can implement the optimal allocation; see e.g. Chari and Kehoe (1999). Here, we focus on the tax instrument most directly related to world interest rate manipulation: taxes on international capital flows.[8]

For expositional purposes, we assume that consumers can only trade one-period bonds on international capital markets, with the home government imposing a proportional tax

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[8]Other tax instruments that could be used to implement the optimal allocation include time-varying trade and consumption taxes (possibly accompanied by production taxes in more general environments). See Jeanne (2011) for a detailed discussion of the equivalence between capital controls and trade taxes.

θ_t on the gross return on net asset position in international bond markets. Standard arguments show that any competitive equilibrium supported by intertemporal trading of consumption claims at date 0 can be supported by trading of one-period bonds. As we discuss later in Section 3.4, none of the results presented here depend on the assumption that one-period bonds are the only assets available.

With only one-period bonds, the per-period budget constraint of the home consumer takes the form

$$q_t a_{t+1} + c_t = y_t + (1 - \theta_{t-1}) a_t - l_t, \tag{9}$$

where a_t denotes the current bond holdings, l_t is a lump sum tax, and $q_t \equiv p_{t+1}/p_t$ is the price of one-period bonds at date t. In addition, consumers are subject to a standard no-Ponzi condition, $\lim_{t\to\infty} p_t a_t \geq 0$. In this environment the home consumer's Euler equation takes the form

$$u'(c_t) = \beta(1 - \theta_t)(1 + r_t)u'(c_{t+1}). \tag{10}$$

where $r_t \equiv 1/q_t - 1$ is the world interest rate. Given a solution $\{c_t\}$ to Home's planning problem (P), the world interest rate is uniquely determined as

$$r_t = \frac{u^{*\prime}(Y - c_t)}{\beta u^{*\prime}(Y - c_{t+1})} - 1, \tag{11}$$

by equations (1) and (3). Thus, given $\{c_t\}$, we can use (10) to construct a unique sequence of taxes $\{\theta_t\}$. We can then set the sequence of assets positions and lump-sum transfers

$$a_t = \sum_{s=t}^{\infty} (p_s/p_t)(c_s - y_s),$$

$$l_t = -\theta_{t-1} a_t,$$

which ensures that the per-period budget constraint (9) and the no-Ponzi condition are satisfied. Since (9), (10), and the no-Ponzi condition are sufficient for optimality it follows that given prices and taxes, $\{c_t\}$ is optimal for the home consumer. This establishes that any solution $\{c_t\}$ of (P) can be decentralized as a competitive equilibrium with taxes.

A positive θ_t can be interpreted as imposing simultaneously a tax θ_t on capital outflows and a subsidy θ_t to capital inflows. Obviously, since there is a representative consumer, only one of the two is active in equilibrium: the outflow tax if the country is a net lender, $a_{t+1} > 0$, and the inflow subsidy if it is a net borrower, $a_{t+1} < 0$. Similarly, a negative θ_t can be interpreted as a subsidy on capital outflows plus a tax on capital inflows.

The bottom line is that $\theta_t > 0$ discourages domestic savings while $\theta_t < 0$ encourages them.

Combining the definition of the wedge (7) with equations (10) and (11), we obtain the following relationship between wedges and taxes on capital flows:

$$\theta_t = 1 - \frac{1 + \tau_t}{1 + \tau_{t+1}}. \tag{12}$$

The previous subsection has already established that variations in domestic consumption c_t along the optimal path are only a function of the current endowment y_t. Since τ_t is only a function of c_t, equation (12) implies that variations in θ_t are only a function of y_t and y_{t+1}. Combining equation (12) with Corollary 1, we then obtain the following result about the structure of optimal capital controls.

Proposition 2 *(Optimal capital flow taxes) Suppose that the optimal policy is implemented with capital flows taxes. Then it is optimal:*

1. *to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if $y_{t+1} > y_t$;*

2. *to tax capital outflows/subsidize capital inflows ($\theta_t > 0$) if $y_{t+1} < y_t$;*

3. *not to distort capital flows ($\theta_t = 0$) if $y_{t+1} = y_t$.*

Proposition 2 builds on the same logic as Proposition 1. Suppose, for instance, that Home is running a trade deficit in periods t and $t+1$. In this case, the home government wants to exercise its monopsony power by lowering domestic consumption in both periods. But if Home grows between these two periods, $y_{t+1} > y_t$, the number of units imported from abroad is lower in period $t+1$. Thus the home government has less incentive to lower consumption in that period. This explains why a tax on capital inflows is optimal in period t: it reduces borrowing in period t, thereby shifting consumption from period t to period $t+1$. The other results follow a similar logic.

It is worth emphasizing that, although the only motive for capital controls in our model is interest rate manipulation, neither the net financial position of Home nor the change in that position are the relevant variables to look at to sign the optimal direction of the tax in any particular period. This is because the effect of a capital flow tax is to affect the relative distortion in consumption decisions between two consecutive periods. Therefore, what matters is whether the monopolistic/monopsonistic incentives to restrict domestic consumption are stronger in period t or $t+1$. In our simple endowment economy, these incentives are purely captured by the growth rate of the endowment, but the same broad principle would extend to more general environments.

Proposition 2 has a number of implications. Consider first an economy that is catching up with the rest of the world in the sense that $y_{t+1} > y_t$ for all t. According to our analysis, it is optimal for this country to tax capital inflows and to subsidize capital outflows. The basic intuition is that a growing country will export more tomorrow than today. Thus it has more incentive to increase export prices in the future, which it can achieve by raising future consumption through a subsidy on capital outflows. For an economy catching up with the rest of the world, larger benefits from future terms-of-trade manipulation are associated with taxes and subsidies that encourage domestic savings.

Consider instead a country that at time t borrows from abroad in anticipation of a temporary boom. In particular, suppose that $y_{t+1} > y_t$ and $y_s = y_t$ for all $s > t+1$. In this situation, the logic of Proposition 2 implies that, at time t, at the onset of the boom, it is optimal to impose restrictions on short-term capital inflows, i.e., to tax bonds with one-period maturity and leave long-term capital inflows unrestricted.[9] This example provides a different perspective on why governments may try to alter the composition of capital flows in favor of longer maturity flows in practice; see Magud, Reinhart, and Rogoff (2011). In our model, incentives to alter the composition of capital flows do not come from the fear of "hot money" but from larger benefits of terms-of-trade manipulation in the short run.

Finally, Proposition 2 has sharp implications for the structure of optimal capital controls in the long-run.

Corollary 2 *(No tax in steady state) In the long run, if endowments converge to a steady state, $y_t \to y$, then taxes on international capital flows converge to zero, $\theta_t \to 0$.*

Corollary 2 is reminiscent of the Chamley-Judd result (Judd, 1985; Chamley, 1986) of zero capital income tax in the long-run. Intuitively, the home government would like to use its monopoly power to influence intertemporal prices to favor the present value of its income. However, at a steady state all periods are symmetric, so it is not optimal to manipulate relative prices. Note that a steady state may be reached with a positive or negative net financial position. Which of these cases applies depends on the entire sequence $\{y_t\}$. Our analysis demonstrates that taxes on international capital flows are unaffected by these long-run relative wealth dynamics. For instance, even if Home, say, becomes heavily indebted, it is not optimal to lower long run interest rates. In our model, even away from a steady state, taxes on international capital flows are determined by the endowments at t and $t+1$ only.

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[9]The tax on two period bonds is easily shown to be $(1 - \theta_t)(1 - \theta_{t+1}) - 1$ and Proposition 1 implies that it is zero in our example.

3.3 An Example with CRRA Utility and Aggregate Fluctuations

Up to now we have focused on the case of a fixed world endowment. Thus we have looked at how optimal capital controls respond to a reallocation of resources between countries, keeping the total pie fixed. This provides a useful benchmark in which all fluctuations in consumption reflect the incentives of the home government to manipulate the world interest rate. Here we show that if domestic and foreign consumers have identical CRRA utility functions, then our results extend to economies with aggregate fluctuations. We also take advantage of this example for a simple exploration of the magnitudes involved with optimal capital controls in terms of quantities and welfare.

Our characterization of the optimal policy of the home government extends immediately to the case of a time-varying world endowment: one just needs to replace Y with Y_t in equation (6). Under the assumption of identical CRRA utility functions, $u(c) = u^*(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma \geq 0$, this leads to a simple relationship between the home share of world endowments, y_t/Y_t, and the home share of world consumption, c_t/Y_t:

$$\left(\frac{c_t/Y_t}{1-c_t/Y_t}\right)^{-\gamma} = \mu \left[1 + \gamma \left(\frac{c_t/Y_t - y_t/Y_t}{1-c_t/Y_t}\right)\right].$$

The left-hand side is decreasing in c_t/Y_t, whereas the right-hand side is increasing in c_t/Y_t and decreasing in y_t/Y_t. Thus the implicit function theorem implies that, along the optimal path, the home share of world consumption, c/Y, is strictly increasing in the home share of world endowments, y/Y. Put simply, if utility functions are CRRA, Proposition 1 generalizes to environments with aggregate fluctuations.

Now consider the wedge τ_t between the marginal utility of domestic and foreign consumption in period t. Under the assumption of CRRA utility functions we have

$$\tau_t = \frac{1}{\mu} \left(\frac{c_t/Y_t}{1-c_t/Y_t}\right)^{-\gamma} - 1.$$

According to this expression, if c/Y is strictly increasing in y/Y along the optimal path, then τ is strictly decreasing. The same logic as in Section 3.2 therefore implies that optimal taxes on capital flows must be such that $\theta_t < 0$ if and only if $y_{t+1}/Y_{t+1} > y_t/Y_t$. In other words, if utility functions are CRRA, Proposition 2 also generalizes to environments with aggregate fluctuations.

As a quantitative illustration of our theory of capital controls as dynamic terms-of-trade manipulation, suppose that foreign endowments $\{y_t^*\}$ are growing at the constant rate $g = 3\%$ per year and that Home is catching up with the rest of the world. To be more
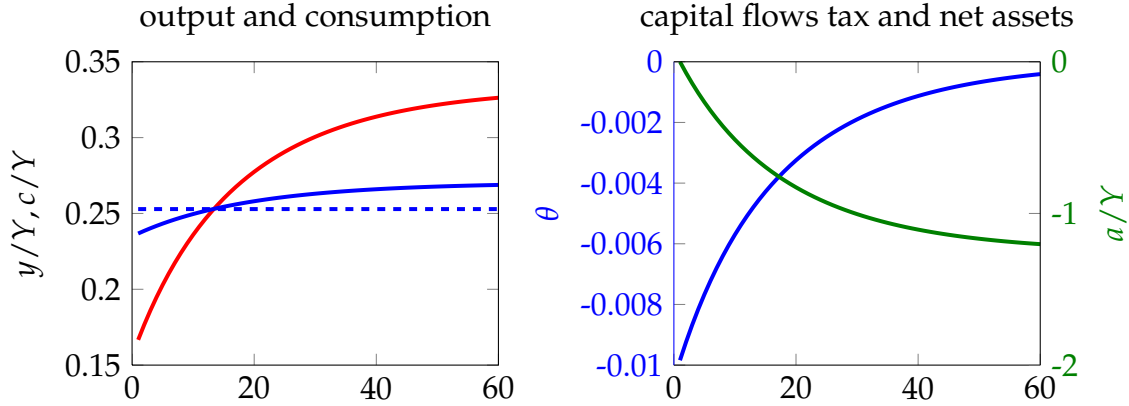
Figure 3: Optimal Allocation and Taxes in a Country Catching Up

Note: In the left panel, the steeper (red) line is the exogenous path for the endowment, the flatter (blue) line is consumption at the optimal policy of the home government, and the dashed line is the efficient no-tax benchmark. In the right panel, the upward sloping (blue) line with axis on the left is the capital flow tax and the downward sloping (green) line with axis on the right is the home assets-to-world-GDP ratio.

specific, suppose that the home endowment is 1/6 of world endowments at date 0 and that it is converging towards being 1/3 in the long run, with the ratio y_t/y_t^* converging to its long run value at a constant speed $\eta = 0.05$.[10]

Figure 3 shows the path of the home share of world endowments and consumption, assuming a unit elasticity of intertemporal substitution, $\gamma = 1$. For comparison, we also plot the path for consumption in the benchmark case with no capital controls. In this case, consistently with consumption smoothing, Home consumes a fixed fraction of the world endowment in all periods. Although optimal capital controls reduce consumption smoothing, intertemporal trade flows are several times larger than domestic output. The optimal tax on capital inflows is less than 1% at date 0 and vanishing in the long run, following the same logic as for Corollary 2. We see that that the optimal tax on capital inflows *decreases* as the value of the home debt *increases*. Compared to the benchmark with no capital controls, optimal taxes are associated with an increase in domestic consumption of 0.12% and a decrease in foreign consumption of 0.07%. Though the welfare impact of optimal capital controls is admittedly not large in this particular example, it is not much smaller than either the estimated gains of international trade or financial integration.[11]

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[10]That is, we assume that

$$y_t/y_t^* - a = (y_0/y_0^* - a)\, e^{-\eta t},$$

with $a = 0.5 > y_0/y_0^* = 0.2$.

[11]According to a fairly large class of trade models, the welfare gains from international trade in the United States are between 0.7% and 1.4% of real GDP; see Arkolakis, Costinot, and Rodríguez-Clare (2009).
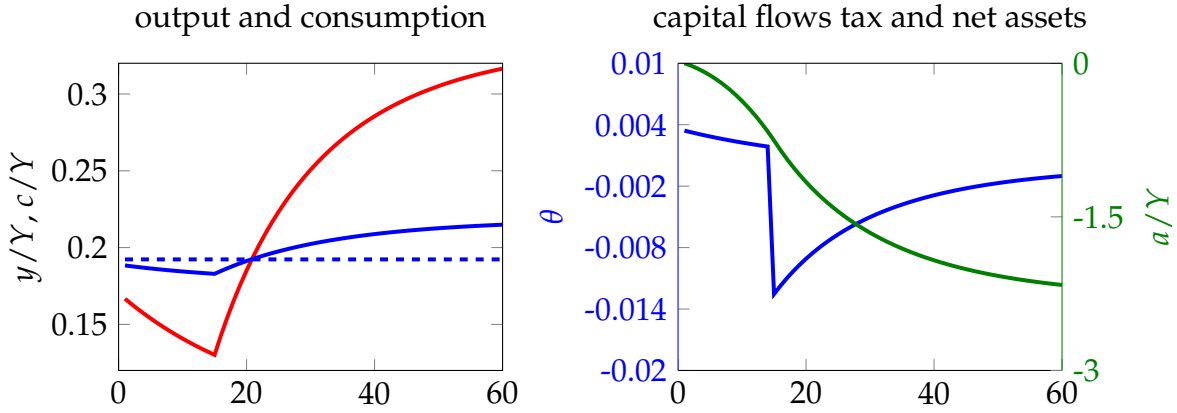
Figure 4: Optimal Allocation and Taxes in a Country Falling Behind Before Catching Up

Note: In the left panel, the steeper (red) line is the exogenous path for the endowment, the flatter (blue) line is consumption at the optimal policy of the home government, and the dashed line is the efficient no-tax benchmark. In the right panel, the non-monotonic (blue) line with axis on the left is the capital flow tax and the downward sloping (green) line with axis on the right is the home assets-to-world-GDP ratio.

Figure 4 considers an alternative endowment path in which Home falls behind in the short-run, before catching up in the long-run. As in the previous example, Home is converging towards having $1/3$ of world endowments with the ratio y_t/y_t^* converging to its long run value at a constant speed $\eta = 0.05$. Because of long-run considerations, we see that Home borrows in all periods. Based on the logic of a two-period model, one might have therefore expected Home to have incentives to tax capital inflows in all periods to reduce domestic borrowing, and in turn, the world interest rate. Yet we see that when falling behind, the home government has incentives to *subsidize* rather than *tax* capital inflows. As discussed earlier, this occurs because capital controls are guided by the relative strength of the desire to to alter the intertemporal price of goods between consecutive periods. In the short-run, the growth rate is negative, hence the subsidy on capital inflows. The pattern of borrowing and lending, per se, is irrelevant.[12]

[12]It should be clear that the reason why a country that borrows may choose to subsidize rather than tax capital inflows is distinct from the reason why a country may find subsidies rather than taxes welfare enhancing in a static model with many goods; see e.g. Feenstra (1986), Itoh and Kiyono (1987), or Bond (1990). In the previous papers, the optimality of subsidies rely on complementarities in demand across goods, which our model with additively separable preferences rules out. Here, imported goods always face a positive wedge, i.e., an import tax, whereas exported goods always face a negative wedge, i.e. an export tax; see equation (8). The reason why a country that borrows may choose to subsidize capital inflows is because taxes on one period bonds are related to, but distinct from static trade taxes; see equation (12).

3.4 Initial Assets, Debt Maturity, and Time-Consistency

So far, we have focused on environments in which: (i) there are no initial assets at date 0 and (ii) one-period bonds are the only assets available. We now briefly discuss how relaxing both assumptions affects our results. We also show that if more debt instruments are available, the optimal allocation is time-consistent: a future government free to choose future consumption, but forced to fulfill previous debt obligations would not want to deviate from the consumption path chosen by its predecessors.

Let $a_{t,s}$ represents holdings at time t of bonds maturing at time s. Suppose the home consumer enters date 0 with initial asset holdings $\{a_{0,t}\}_{t=0}^{\infty}$. The asset holdings now enter the intertemporal budget constraints of the home and foreign consumers. In particular, the budget constraint of the foreign consumer generalizes to

$$\sum_{t=0}^{\infty} p_t(c_t^* - y_t^* - a_{0,t}^*) = 0,$$

where $a_{0,t}^* = -a_{0,t}$ denotes initial asset holdings abroad. The other equilibrium conditions are unchanged, so Home's planning problem becomes

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{P_0}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)(c_t - y_t + a_{0,t}^*) = 0. \tag{13}$$

Compared to the case without initial assets, the only difference is the new implementability constraint (13) that depends on $\{y_t - a_{0,t}^*\}$ rather than $\{y_t\}$. Accordingly, Proposition 1 and Corollary 1 simply generalize to environments with initial assets $\{a_{0,t}^*\}_{t=0}^{\infty}$ provided that they are restated in terms of changes in $y_t - a_{0,t}^*$ rather than changes in y_t.

Throughout our analysis, we have assumed that the home government can freely commit at date 0 to a consumption path $\{c_t\}$. Now that we have recognized the role of the initial asset positions, this assumption may seem uncomfortably restrictive. After all, along the optimal path, the debt obligations $\{a_{t,s}^*\}_{s=t}^{\infty}$ held at date t will typically be different from the obligations $\{a_{0,s}^*\}_{s=t}^{\infty}$ held at date 0. Accordingly, a government at later dates may benefit from deviating from the consumption chosen at date 0.

We now demonstrate that this is not the case if the government has access to bonds

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Specifically, for a country that borrows, a subsidy on one-period bonds, $\theta_t > 0$, is equivalent to a time-varying import tax, $\tau_{t+1} > \tau_t > 0$; it does not require import subsidies at any point in time.

of arbitrary maturity. The basic idea builds on the original insight of Lucas and Stokey (1983). At any date t, the foreign consumer is indifferent between many future asset holdings $\{a_{t+1,s}^*\}_{s=t+1}^\infty$. Given a consumption sequence $\{c_t^*\}$ that maximizes her utility subject to her budget constraint, she is indifferent between any bond holdings satisfying

$$\sum_{s=t+1}^\infty p_s(c_s^* - y_s^* - a_{t+1,s}^*) = 0. \tag{14}$$

As we show in the appendix, this degree of freedom is sufficient to construct sequences of debt obligations $\{a_{t,s}^*\}_{s=t}^\infty$ for all $t \geq 1$ such that the solution of

$$\max_{\{c_s\}} \sum_{s=t}^\infty \beta^s u(c_s)$$

subject to

$$\sum_{s=t}^\infty \beta^s u^{*\prime}(Y - c_s)(c_s - y_s + a_{t,s}^*) = 0 \tag{15}$$

coincides with the solution of (P_0) at all dates $t \geq 0$. In short, if the home government can enter debt commitments at all maturities, the optimal allocation derived in Section 3.1 is time-consistent.

4   Intertemporal and Intratemporal Trade

How do the incentives to tax capital flows change in a world with many goods? In a one-good economy, the only form of terms-of-trade manipulation achieved by taxing capital flows is to manipulate the world interest rate. In a world with many goods, distorting the borrowing and lending decisions of domestic consumers also affects the relative prices of the different goods traded in each period. In this section, we explore how these new intratemporal considerations change optimal capital flow distortions.

In order to maintain the focus of our analysis on optimal capital controls, we proceed under the assumption that Home is constrained by an international free-trade agreement that prohibits good specific taxes/subsidies in all periods. As in the previous section, Home is still allowed to impose taxes on capital flows that distort intertemporal decisions. This means that while Home cannot control the path of consumption of each specific good i, it can still control the path of aggregate consumption. As we shall see, in general, the path of aggregate consumption can affect relative prices at any point in time, thus creating additional room for terms-of-trade manipulation, even for countries that cannot affect the

world interest rate.

4.1 The Monopolist Problem Revisited

The basic environment is the same as in Section 2.1, except that there are $n > 1$ goods. Thus domestic consumption and output, c_t and y_t, are now vectors in R_+^n. We assume that the domestic consumer has additively separable preferences represented by

$$\sum_{t=0}^{\infty} \beta^t U(C_t),$$

where U is increasing and strictly concave, $C_t \equiv g(c_t)$ is aggregate domestic consumption at date t, and g is increasing, concave, and homogeneous of degree one. Analogous definitions apply to U^* and $C_t^* \equiv g^*(c_t^*)$.

In the absence of taxes, the intertemporal budget constraint of the home consumer is now given by

$$\sum_{t=0}^{\infty} p_t \cdot (c_t - y_t) \leq 0,$$

where $p_t \in R_+^n$ denotes the intertemporal price vector for period-t goods and $\cdot$ is the inner product. A similar budget constraint applies in Foreign.

As in Section 2.2, we use the primal approach to characterize the optimal policy of the home government. In this new environment, the home government's objective is to set consumption $\{c_t\}$ in order to maximize the lifetime utility of its representative consumer subject to (i) utility maximization by the foreign representative consumer at (undistorted) world prices p_t; (ii) market clearing in each period; and (iii) a free trade agreement that rules out good specific taxes or subsidies.

Constraint (i) can be dealt with as we did in the one-good case. In vector notation, the first-order conditions associated with utility maximization by the foreign consumer generalize to

$$\beta^t U^{*\prime}(C_t^*) g_c^*(c_t^*) = \lambda^* p_t, \tag{16}$$

$$\sum_{t=0}^{\infty} p_t \cdot (c_t^* - y_t^*) = 0. \tag{17}$$

Next, note that if Home cannot impose good specific taxes or subsidies, the relative price of any two goods i and j in period t, p_{it}/p_{jt}, must be equal in the two countries and equal to the marginal rates of substitution $g_i(c_t)/g_j(c_t)$ and $g_i^*(c_t^*)/g_j^*(c_t^*)$. Accordingly, the

consumption allocation (c_t, c_t^*) in any period t is Pareto efficient and solves

$$C^* (C_t) = \max_{c, c^*} \{g^*(c^*) \quad \text{subject to} \quad c + c^* = Y \text{ and } g(c) \geq C_t\} \quad (18)$$

for some C_t. Therefore, constraints (ii) and (iii), can be captured by letting Home choose an aggregate consumption level C_t, which identifies a point on the static Pareto frontier. The consumption vectors at time t are then given by the corresponding solutions to problem (18), which we denote by $c\,(C_t)$ and $c^*\,(C_t)$.

We can then state Home's planning problem in the case of many goods as

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad (P')$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \rho(C_t) \cdot (c\,(C_t) - y_t) = 0, \quad (19)$$

where $\rho(C_t) \equiv U^{*\prime}\,(C^*(C_t))\,\nabla g^*(c^*\,(C_t))$. Equation (19) is the counterpart of the implementability constraint in Section 2.2. In line with our previous analysis, we assume that $\rho(C_t) \cdot (c\,(C_t) - y_t)$ is a strictly convex function of C_t. This implies the uniqueness of the solution to (P').

4.2 Optimal Allocation

With many goods, the first-order condition associated with Home's planning problem generalizes to

$$U'\,(C_t) = \mu \left[\rho(C_t) \cdot \frac{\partial c(C_t)}{\partial C_t} + \frac{\partial \rho(C_t)}{\partial C_t} \cdot (c(C_t) - y_t) \right], \quad (20)$$

where μ still denotes the Lagrange multiplier on the implementability constraint. Armed with condition (20), we can now follow the same strategy as in the one-good case. First we will characterize how $\{C_t\}$ covaries with $\{y_t\}$ along the optimal path. Second we will derive the associated implications for the structure of optimal capital controls.

The next proposition describes the relationship between domestic consumption and domestic endowments along the optimal path.

Proposition 3 *(Procyclical aggregate consumption) Suppose that between periods t and $t+1$ there is a small change in the home endowment $dy_{t+1} = y_{t+1} - y_t$. Then the home consumption is higher in period $t+1$, $C_{t+1} > C_t$, if and only if $\partial \rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$.*

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In the one good case, $\partial\rho(C_t)/\partial C_t$ simplifies to $-u^{*\prime\prime}(Y - c_t)$, which is positive by the concavity of u^*. Therefore, whether domestic consumption grows or not only depends on whether the level of domestic endowments is increasing or decreasing. In the multi-good case, by contrast, this also depends on the composition of domestic endowments and on how relative prices respond to changes in C_t.

In order to highlight the importance of these compositional effects, in an economy with many goods, consider the effect of a small change in domestic endowment that leaves its market value unchanged at period t prices. That is, suppose $\rho(C_t) \cdot dy_{t+1} = 0$. In the one good case this can only happen if the endowment level does not change, thereby leading to a zero capital flow tax. In the multi-good case this is no longer true. According to Proposition 3, consumption would grow if and only if

$$\mathrm{Cov}\left(\frac{\rho_i'(C_t)}{\rho_i(C_t)}, \rho_i(C_t)dy_{it+1}\right) > 0.$$

Here, what matters is whether the composition of endowments tilts towards goods that are more or less price sensitive to changes in C_t. We will come back to the role of this compositional effects in more detail in Section 4.4.

4.3   Optimal Taxes on International Capital Flows

In line with Section 3.2, let us again assume that consumers can only trade one-period bonds on international capital markets. But compared to Section 3.2, suppose now that there is one bond for each good. Since the home government cannot impose good specific taxes/subsidies, it must impose the same proportional tax θ_t on the gross return on net lending in all bond markets. So the per period budget constraint of the domestic consumer takes the form

$$p_{t+1} \cdot a_{t+1} + p_t \cdot c_t = p_t \cdot y_t + (1 - \theta_{t-1})(p_t \cdot a_t) - l_t,$$

where $a_t \in R_+^n$ now denotes the vector of current asset positions and l_t is a lump sum tax. As before, the domestic consumer is subject to the no-Ponzi condition, $\lim_{t\to\infty} p_t \cdot a_t \geq 0$. The first-order conditions associated with utility maximization at home are given by

$$U'(C_t)g_i(c_t) = \beta(1 - \theta_t)(1 + r_{it})U'(C_{t+1})g_i(c_{t+1}), \text{ for all } i = 1, ..., n. \quad (21)$$

where $r_{it} \equiv p_{it}/p_{it+1} - 1$ is a good-specific interest rate. Let $P_t \equiv \min_c \{p_t \cdot c : g(c) \geq 1\}$ denote the home consumer price index at date t. Using this notation, the previous condi-

tions can be rearranged in a more compact form as

$$U'(C_t) = \beta(1-\theta_t)(1+R_t)U'(C_{t+1}), \tag{22}$$

where $R_t \equiv P_t/P_{t+1} - 1$ is the home real interest rate at date t.[13] Since there are no taxes abroad, the same logic implies

$$U^{*\prime}(C_t^*) = \beta(1+R_t^*)U^{*\prime}(C_{t+1}^*), \tag{23}$$

where $R_t^* \equiv P_t^*/P_{t+1}^* - 1$ is the foreign real interest rate at date t. Equations (22) and (23) are the counterparts of the Euler equations (10) and (11) in the one-good case. Combining these two expressions we obtain

$$\theta_t = 1 - \frac{U'(C_t)}{U^{*\prime}(C_t^*)} \frac{U^{*\prime}(C_{t+1}^*)}{U'(C_{t+1})} \frac{(1+R_t^*)}{(1+R_t)}.$$

If we follow the same approach as in the one-good case and let $\tau_t \equiv U'(C_t)/\mu U^{*\prime}(C_t^*) - 1$ denote the wedge between the marginal utility of domestic and foreign consumption, we can rearrange Home's tax on international capital flows as

$$\theta_t = 1 - \left(\frac{1+\tau_t}{1+\tau_{t+1}}\right)\left(\frac{P_{t+1}/P_{t+1}^*}{P_t/P_t^*}\right).$$

With many goods, the sign of θ_t depends on (i) whether the wedge τ_t between the marginal utility of domestic and foreign consumption is increasing or decreasing and (ii) whether Home's real exchange rate, P_t/P_t^*, appreciates or depreciates between t and $t+1$. Like in the one-good case, one can check that the wedge is a decreasing function of home aggregate consumption C_t. In the next proposition we further demonstrate that an increase in C_t is always associated with an appreciation of Home's real exchange rate. Combining these two observations with Proposition 3, we obtain the following result.

Proposition 4 *(Optimal capital flow taxes revisited) Suppose that the optimal policy is implemented with capital flow taxes and that between periods t and t + 1 there is a small change in the home endowment $dy_{t+1} = y_{t+1} - y_t$. Then it is optimal:*

1. *to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$;*

2. *to tax capital outflows/subsidize capital inflows ($\theta_t > 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} < 0$;*

---

[13] In the proof of Proposition 4 in the appendix, we formally establish that $P_t = p_{it}/g_i(c_t)$ for all $i = 1, ..., n$. Equation (22) directly derives from this observation and equation (21).

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3. *not to distort capital flows* ($\theta_t = 0$) *if* $\partial \rho(C_t)/\partial C_t \cdot dy_{t+1} = 0$.

In order to understand better how intertemporal and intratemporal considerations affect the structure of the optimal tax schedule, let us decompose the price vector in period t into an intertemporal price and an intratemporal vector of relative prices: $p_t = P_t^* \pi_t$, where $\pi_{it} \equiv p_{it}/P_t^*$ denotes the price of good i in terms of foreign consumption at date t.

Using the previous decomposition, we see that the sign of the expression $\partial \rho(C_t)/\partial C_t \cdot dy_{t+1}$ in Proposition 4 is the same as the sign of the following expression[14]

$$\frac{P^{*\prime}(C_t)}{P^*(C_t)} \sum_i \pi_i(C_t) \, dy_{it+1} + \sum_i \frac{\pi_i'(C_t)}{\pi_i(C_t)} \pi_i(C_t) \, dy_{it+1}. \tag{24}$$

The first term captures the intertemporal price channel and is proportional to the change in the value of output. It is possible to show that $P^{*\prime}(C_t) > 0$. Thus an increase in the value of home output—all else equal—pushes in the direction of a tax on capital inflows/subsidy to capital outflows. This follows the same logic as in the one-good case.

The new element is the second term in (24), which captures intratemporal terms-of-trade effects. The sign of this term depends on the elasticity of relative prices to changes in domestic consumption. To sign this term we need to know more about preferences. The simplest case is the case of symmetric preferences in which g and g^* are the same. In that case, the Pareto set in the Edgeworth box is a straight line and relative prices are independent of the point we choose (i.e., of C_t). Not surprisingly, in this case the analysis boils down to the one-good case. Therefore, the interesting case is the case of asymmetric preferences, which we now turn to.

4.4 An Example with CRRA and Asymmetric Cobb-Douglas Utility

In this subsection we focus on a simple example in which the effects of intratemporal considerations can be captured analytically. There are two goods. The upper-level utility function at home is CRRA and the lower-level utility is Cobb-Douglas:

$$U(C) = \frac{1}{1-\gamma} C^{1-\gamma}, \quad C = c_1^{\alpha} c_2^{1-\alpha}, \tag{25}$$

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[14]Just notice that

$$\rho_t = \lambda^* \beta^{-t} p_t = \lambda^* \beta^{-t} P_t^* \pi_t,$$

from the optimality condition of the foreign consumer and so

$$\frac{\rho_i'(C_t)}{\rho_i(C_t)} = \frac{P^{*\prime}(C_t)}{P^*(C_t)} + \frac{\pi_i'(C_t)}{\pi_i(C_t)}.$$

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where $\gamma \geq 0$ and $\alpha > 1/2$. Foreign utility functions take the same form, but the roles of goods 1 and 2 are reversed

$$U^* (C^*) = \frac{1}{1 - \gamma} (C^*)^{1-\gamma}, C^* = (c_2^*)^\alpha (c_1^*)^{1-\alpha}. \tag{26}$$

Since $\alpha > 1/2$, Home has a higher relative demand for good 1 in all periods. Without risk of confusion, we now refer to good 1 and good 2 as Home's "import-oriented" and "export-oriented" sectors, respectively. The next proposition highlights how this distinction plays a key role in linking intertemporal and intratemporal terms-of-trade motives.[15]

Proposition 5 *(Import- versus export-oriented growth) Suppose that equations (25)-(26) hold with $\gamma \geq 0$ and $\alpha > 1/2$ and that between periods t and $t + 1$ there is a small change in the home endowment $dy_{t+1} = y_{t+1} - y_t$. If growth is import-oriented, $dy_{1t+1} > 0$ and $dy_{2t+1} = 0$, it is optimal to tax capital inflows/subsidize capital outflows ($\theta_t < 0$). Conversely, if growth is export-oriented, $dy_{1t} = 0$ and $dy_{2t+1} > 0$, it is optimal to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if and only if $\gamma > \left(\frac{2\alpha-1}{\alpha} \right) \left(\frac{P_t^* C_t^*}{P_t^* C_t^* + P_t C_t} \right)$.*

The idea behind the first part of Proposition 5 is closely related to Proposition 2. In periods in which Home controls a larger fraction of the world endowment of good 1, the incentive to subsidize consumption C increases. Here, however, the reason is twofold. First, a larger endowment of good 1 means that Home is running a smaller (net) trade deficit, which reduces the incentive to depress the intertemporal price P^*. Second, it means that within the period the country is selling more of good 1. Since home preferences are biased towards good 1, an increase in C drives up the intratemporal price of good 1, which further increases the incentives to subsidize aggregate consumption.[16]

By contrast, when endowment growth is export-oriented, intertemporal and intratemporal considerations are not aligned anymore. If the elasticity of intertemporal substitution, $1/\gamma$, is low enough, the intertemporal motive for terms-of-trade manipulation dominates and we get the same result as in the one good economy. If instead that elasticity is high enough, the result goes in the opposite direction. Namely, it is possible that

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[15]Another simple example that can be solved analytically is the case of tradable and non-tradable goods. If there is only one tradable good, then Proposition 2 applies unchanged to changes in the endowment of the tradable good. The only difference between this case and the one-good case studied in Section 3 is that taxes on capital inflows/subsidies on capital outflows ($\theta_t < 0$) now are always accompanied by a real exchange rate appreciation, whereas taxes on capital outflows/subsidies on capital inflows ($\theta_t > 0$) now are always accompanied by a real exchange rate depreciation.

[16]Like in Section 3.1, whether Home is a net seller or a net buyer of good 1 does not matter per se. What matters for the sign of optimal taxes is the fact that larger endowments of good 1 at date $t + 1$ imply that Home tends to sell more (or buy less) of that good than at date t and, in turn, to benefit more (or to lose less) from an increase in the price of that good.
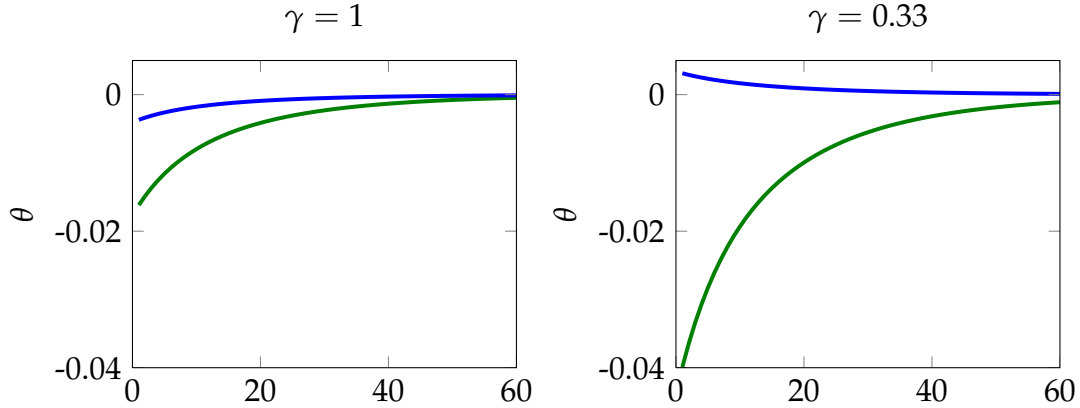
$\gamma = 1$ $\gamma = 0.33$

Figure 5: Optimal Taxes under Import- and Export-Oriented Scenarios

Note: In both panels, the bottom (green) line is the optimal tax on capital flows under the import-oriented scenario (growth in sector 1) and the top (blue) line is the optimal tax on capital flows under the export-oriented scenario (growth in sector 2).

when Home receives a larger endowment of good 2, it decides to subsidize aggregate consumption less, even though the increase in y_2 is reducing its (net) trade deficit. Intuitively, Home now benefits from reducing its own consumption since this increases the intratemporal price of good 2 due again to the fact that, relative to foreign preferences, home preferences are biased towards good 1. Proposition 5 formally demonstrates that the intratemporal terms-of-trade motive is more likely to dominate the intertemporal one if demand differences between countries are large and/or Foreign accounts for a large share of world consumption.

In order to illustrate the quantitative importance of this effect, we return to the exercise presented in Section 3.3 in which Home is catching up with the rest of the world. For simplicity, the world endowments of both goods are assumed to be constant over time. In the first panel of Figure 5, the intertemporal elasticity of substitution is set to unity, $\gamma = 1$, and demand differences are set such that $\alpha = 3/4$. The bottom (green) curve represents the optimal tax on capital flows in the import-oriented scenario: the home endowment of good 2 is fixed, but the home endowment of good 1 is $1/6$ of world endowments at date 0 and is converging towards being $1/3$ in the long run, with the ratio y_{1t}/y_{1t}^* converging to its long run value at a constant speed $\eta = 0.05$. The top (blue) curve instead represents the optimal tax on capital flows in the export-oriented scenario: the home endowment of good 1 is fixed, but the home endowment of good 2 is growing. In order to make the two scenarios comparable, the growth rate of good 2's endowments is chosen such that the home share of world income in all periods is the same as in the import-oriented scenario.

In all periods we see that the optimal tax on capital inflows is lower in the export-oriented scenario. While taxes converge to zero under both scenarios, the tax on capital inflows at date 0 is four times larger in the import-oriented scenario than in the export-oriented one: 1.6% versus 0.4%. In the second panel of Figure 5, we repeat the same experiments under the assumption that $\gamma = 0.33$. In this situation, the intratemporal terms-of-trade motives now dominate the intertemporal ones under the export-oriented scenario. When there is growth at home relative to the rest of the world, but growth is concentrated in sector 2, Home finds it optimal to subsidize rather than tax capital inflows. At date 0, the optimal *subsidy* on capital inflows is now around 0.4%.

4.5   Capital Controls in a Small-Open Economy

In Section 3, the only motive for capital controls was the manipulation of world interest rates. While such motive may be relevant for large countries, many small open economies that use capital controls in practice are unlikely to have significant effects on world interest rates. The goal of this final subsection is to illustrate how, because of the interaction between intertemporal and intratemporal trade, terms-of-trade motives may still make capital controls optimal for such economies.

Consider an economy with two goods. In line with the previous section, suppose that the upper-level utility function at Home is CRRA and the lower-level utility is Cobb-Douglas in both countries:

$$U(C) = \frac{1}{1-\gamma} C^{1-\gamma}, C = c_1^{1/2} c_2^{1/2},$$

$$U^*(C^*) = \frac{N-1}{1-\gamma}(C^*)^{1-\gamma}, C^* = \frac{c_1^{*1/N} c_2^{*1-1/N}}{N-1},$$

where $\gamma \geq 0$. World endowments of good 1 are equal to Y_1, whereas world endowments of good 2 are equal to NY_2. One can think of this economy as the reduced-form of a more general environment in which there are N countries in the world; each country is endowed with a differentiated good; and each country spends a constant fraction of its income on its own good as well as a CES aggregator of all goods in the world economy.

In the appendix we show that as N goes to infinity, i.e. as Home becomes a "small" open economy, Home's planning problem converges towards

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \qquad (P_S)$$

28

subject to

$$\sum_{t=0}^{\infty} \beta^t Y_2^{-\gamma} \left[\frac{g_1^*(c^*(C_t))}{g_2^*(c^*(C_t))} (c_1(C_t) - y_{1t}) + (c_2(C_t) - y_{2t}) \right] = 0, \tag{27}$$

where $g_1^*(c^*(C_t))/g_2^*(c^*(C_t)) = Y_2 + \frac{1}{2} \left[C_t^2/Y_1 + \sqrt{(C_t^2/Y_1)^2 + 4Y_2(C_t^2/Y_1)} \right]$. In the limit, aggregate consumption abroad, C_t^*, converges towards Y_2 in all periods, independently of Home's aggregate consumption decision. From the foreign consumer's Euler equation (23), the world real interest rate R_t^* therefore converges towards $R^* = 1/\beta - 1$ in all periods. Accordingly, Home cannot manipulate its intertemporal terms-of-trade. Yet, as equation (27) illustrates, Home can still manipulate its intratemporal temporal terms-of-trade: $g_1^*(c^*(C_t))/g_2^*(c^*(C_t))$ is strictly increasing in Home's aggregate consumption, C_t. As a result, Home will depart from perfect consumption smoothing along the optimal path. Since departures from perfect consumption smoothing along the optimal path can be implemented using taxes on capital flows, this establishes that in a neoclassical benchmark model in which terms-of-trade manipulation is the only motive for capital controls, even a country that cannot affect world interest rates may have incentives to tax international capital flows.

In this example, a small-open economy accounts for an infinitesimal fraction of aggregate consumption in every period. Thus it cannot affect intertemporal prices. Yet, it always accounts for a signification fraction of the consumption of one of the two goods. Thus it can, and will want to, affect intratemporal prices. If good specific trade taxes and subsidies are prohibited by international agreements, capital controls offer an alternative way to achieve that goal.

5 Capital Control Wars

In this section we go back to the one-good case, but consider the case in which both countries set capital controls optimally, taking as given the capital controls chosen by the other country. As before, we assume that consumers can only trade one-period bonds on international capital markets, but we now let both the home and foreign government impose proportional taxes θ_t and θ_t^*, respectively, on the gross return on net asset position in international bond markets. At date 0, we assume that the two governments simultaneously choose the sequences $\{\theta_t\}$ and $\{\theta_t^*\}$ and commit to them. Given this assumption, we can use the same primal approach developed in previous sections to offer a first look at the

outcome of capital control wars.[17]

5.1 Nash Equilibrium

We look for a Nash equilibrium, so we look at each government's optimization problem taking the other government's tax sequence as given. Focusing on the problem of the home government, the optimal taxes can be characterized in terms of a planning problem involving directly the quantities consumed, as in the unilateral case. Given the sequence $\{\theta_t^*\}$ the foreign consumer's Euler equation can be written as

$$u^{*\prime}(c_t^*) = \beta(1 - \theta_t^*)(1 + r_t)u^{*\prime}(c_{t+1}^*). \tag{28}$$

Since $1 + r_t = p_t/p_{t+1}$, a standard iterative argument then implies

$$p_t = \beta^t \left[\prod_{s=0}^{t-1}(1 - \theta_s^*)\right]\left[p_0 u^{*\prime}(c_t^*)/u^{*\prime}(c_0^*)\right].$$

Accordingly, Home's planning problem is now given by

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{P_N}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)\left[\prod_{s=0}^{t-1}(1 - \theta_s^*)\right](c_t - y_t) = 0,$$

where the new implementability constraint captures the fact that the home government now takes foreign capital flow taxes as given. This yields the optimality condition

$$u'(c_t) = \mu \left[\prod_{s=0}^{t-1}(1 - \theta_s^*)\right]\left[u^{*\prime}(c_t^*) - u^{*\prime\prime}(c_t^*)(c_t - y_t)\right], \tag{29}$$

which further implies

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1}{(1 - \theta_t^*)}\frac{u^{*\prime}(c_t^*) - u^{*\prime\prime}(c_t^*)(c_t - y_t)}{u^{*\prime}(c_{t+1}^*) - u^{*\prime\prime}(c_{t+1}^*)(c_{t+1} - y_{t+1})}. \tag{30}$$

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[17]The assumption of commitment is stronger here than in previous sections since it also precludes countries to respond to the other country's policies as they unfold over time. This de facto rules out any equilibrium in which governments may choose to cooperate along the equilibrium path, e.g., to have zero taxes on capital controls, by fear of being punished if they were to deviate from the equilibrium strategies. See Dixit (1987) for an early discussion of related issues in a trade context.

From the domestic consumer's Euler equation, we also know that

$$u'(c_t) = \beta(1 - \theta_t)(1 + r_t)u'(c_{t+1}). \tag{31}$$

Combining equations (30) and (31) with equation (28), we obtain after simplifications

$$1 - \theta_t = \frac{1 - \frac{u^{*\prime\prime}(c_t^*)}{u^{*\prime}(c_t^*)}(c_t - y_t)}{1 - \frac{u^{*\prime\prime}(c_{t+1}^*)}{u^{*\prime}(c_{t+1}^*)}(c_{t+1} - y_{t+1})}.$$

The planning problem of the foreign government is symmetric. So the same logic implies

$$1 - \theta_t^* = \frac{1 - \frac{u''(c_t)}{u'(c_t)}(c_t^* - y_t^*)}{1 - \frac{u''(c_{t+1})}{u'(c_{t+1})}(c_{t+1}^* - y_{t+1}^*)}.$$

Substituting for the foreign tax on international capital flows in equation (30) and using the good market clearing condition (3), we obtain

$$\frac{u'(c_t) + u''(c_t)(c_t - y_t)}{u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t)} = \frac{u'(c_{t+1}) + u''(c_{t+1})(c_{t+1} - y_{t+1})}{u^{*\prime}(Y - c_{t+1}) - u^{*\prime\prime}(Y - c_{t+1})(c_{t+1} - y_{t+1})},$$

which can be rearranged as

$$\frac{u'(c_t) + u''(c_t)(c_t - y_t)}{u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t)} = \alpha, \text{ for all } t \geq 0, \tag{32}$$

where $\alpha \equiv [u'(c_0) + u''(c_0)(c_0 - y_0)]/[u^{*\prime}(Y - c_0) - u^{*\prime\prime}(Y - c_0)(c_0 - y_0)] > 0$. This is the counterpart of equation (6) in Section 2. In particular, using equations (29) and (31) and their counterparts in Foreign, one can check that $\alpha = \lambda\mu^*/\lambda^*\mu$, where λ and λ^* are the Lagrange multipliers associated with the intertemporal budget constraints in both countries.

The next lemma provides sufficient conditions under which a Nash equilibrium exists.

Lemma 1 *Suppose that the following conditions hold:* (i) *u and u^* are twice continuously differentiable, strictly increasing, and strictly concave, with $\lim_{c \to 0} u'(c) = \lim_{c^* \to 0} u'(c^*) = \infty$;* (ii) *$u'(c_t)(c_t - y_t)$ and $u^{*\prime}(c_t^*)(c_t^* - y_t^*)$ are strictly increasing and strictly concave in c_t and c_t^*, respectively, for all y_t and y_t^*; and* (iii) *y_t and y_t^* are bounded away from zero for all t. Then a Nash Equilibrium exists.*

Compared to Section 3, the new condition being imposed is that $u'(c_t)(c_t - y_t)$ and $u^{*\prime}(c_t^*)(c_t^* - y_t^*)$ are strictly increasing in c_t and c_t^*, respectively. In the case of unilaterally

optimal capital controls, this condition necessarily holds locally; see equation (6). If not, a country could simultaneously increase consumption and loosen the implementability constraint. To establish existence of a Nash equilibrium we now require this condition to hold globally. As we next demonstrate, this new condition and our previous assumptions are also sufficient for consumption to be procyclical along the Nash equilibrium.

5.2 Main Results Revisited

Intuitively, one may expect that an increase in y would necessarily lead to an increase in c. Indeed, we have established in Section 3 that if Home were to impose taxes unilaterally, it would like to increase c in response to a positive shock in y. The same logic implies that if Foreign were to impose taxes unilaterally, it would like to decrease c^*, i.e., to increase c as well, in response to a positive shock in y. Thus both unilateral responses point towards an increasing relationship between c and y. As the next lemma demonstrates, if the assumptions of Lemma 1 are satisfied, the previous intuition is correct.

Lemma 2 *Suppose that the assumptions of Lemma 1 hold. Then for any two periods t and s, if the home endowment is larger in s, $y_s > y_t$, then the home consumption is also higher, $c_s > c_t$.*

Using the procyclicality of consumption along the Nash equilibrium, we can use the domestic and foreign consumer's Euler equations to characterize capital control wars the same way we characterized optimal capital controls in Section 3. Our main result about capital control wars can be stated as follows.

Proposition 6 *(Capital control wars) Suppose that the assumptions of Lemma 1 hold. Then along the Nash equilibrium, the home and foreign capital flow taxes are such that:*

1. *Home interest rates are higher than foreign interest rates ($\theta_t < \theta_t^*$) if $y_{t+1} > y_t$;*

2. *Home interest rates are lower than foreign interest rates ($\theta_t > \theta_t^*$) if $y_{t+1} < y_t$;*

3. *Home and foreign interest rates are equal ($\theta_t = \theta_t^*$) if $y_{t+1} = y_t$.*

If there are no intertemporal distortions abroad, $\theta_t^* = 0$, then like in Section 3, an increase in domestic endowments, $y_{t+1} > y_t$, leads to a tax on capital inflows or a subsidy to capital outflows, $\theta_t < 0$, which is associated with higher domestic interest rates, $(1 - \theta_t)(1 + r_t) > 1 + r_t$. In general, however, we cannot sign θ_t and θ_t^*. The intuition for this result is a combination of the intuition for the unilateral policy of Home and Foreign. Suppose, for instance, that Home is running a trade deficit in period t. An increase in the
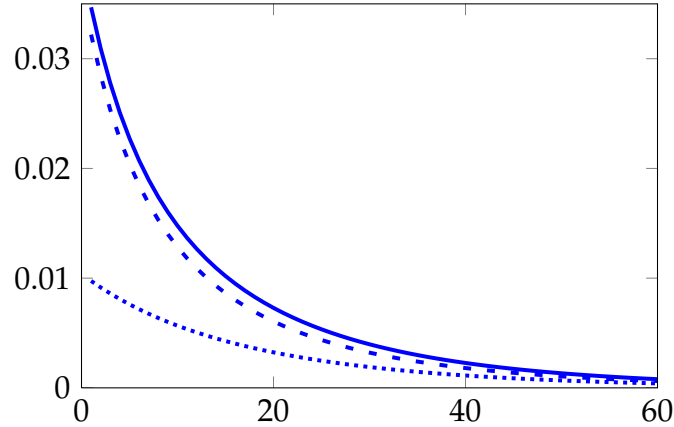
Figure 6: Interest rate differentials with capital control wars and unilaterally optimal taxes

Note: The solid line represents the Nash equilibrium interest rate differential (as a percentage of the world return to net lending), while the dotted and dashed lines show the interest rate differentials under the unilaterally optimal capital flow taxes for Home and Foreign, respectively.

home endowment reduces the trade deficit and reduces the incentives of the home government to repress domestic consumption. Foreign incentives are symmetric, meaning that the foreign government has less incentives to stimulate foreign consumption. The increase in domestic consumption and the reduction in foreign consumption between periods t and $t + 1$ can be achieved in two ways: by a tax on capital inflows at home, $\theta_t < 0$, or by a tax on capital outflows abroad, $\theta_t^* > 0$. Because of the general equilibrium response of world prices, we do not necessarily need both. All we need is that $\theta_t < \theta_t^*$, i.e. that domestic interest rates are higher than foreign interest rates.

To conclude, we compare the Nash equilibrium capital flow taxes to the unilaterally optimal taxes for both countries, i.e. the best response to a zero tax, using again the parameterized example presented in Section 3.3. In Figure 6, we see that a capital control war leads to a larger interest rate differential between the two countries (as a percentage of the world return to net lending) than either one of the two unilateral outcomes. Far from canceling each other out, the net distortion on capital flows is therefore larger when both countries set capital controls optimally. Compared to the benchmark with no capital controls, a capital control war here decreases consumption by 0.49% in the country catching-up (Home) and by 0.05% in the rest of the world (Foreign). Interestingly, even though the interest rate differential is close to its value when Foreign sets capital controls unilaterally, both countries are worse off in the Nash equilibrium. In this particular

example, neither country wins the capital control war.[18]

6 Concluding Remarks

In this paper we have developed a theory of capital controls as dynamic terms-of-trade manipulation. We have studied an infinite horizon endowment economy with two countries where one country chooses taxes on international capital flows in order to maximize the welfare of its representative agent, while the other country is passive. We have shown that capital controls are not guided by the absolute desire to alter the intertemporal price of the goods produced in any given period, but rather by the relative strength of this desire between two consecutive periods. Specifically, it is optimal for the strategic country to tax capital inflows (or subsidize capital outflows) if it grows faster than the rest of the world and to tax capital outflows (or subsidize capital inflows) if it grows more slowly. In the long-run, if relative endowments converge to a steady state, taxes on international capital flows converge to zero. Although our theory of capital controls emphasizes interest rate manipulation, the pattern of borrowing and lending, per se, is irrelevant.

With many goods, we have shown that optimal capital controls depend both on the level and composition of growth across goods. If countries have different preferences, a change in the time profile of consumption not only affects the interest rate but also the relative prices of consumption goods in each given period. Accordingly, even if all static trade distortions are banned by a free-trade agreement, away from steady state, intratemporal prices may not be at their undistorted levels if capital controls are allowed. Finally we have studied capital control wars in which the two countries simultaneously set taxes on capital flows. In the simple quantitative example that we consider, far from canceling each other out, the net distortion on capital flows is larger than in the unilateral case.

Our theory of capital controls is unapologetically normative. It does not try to explain the past behaviors of particular governments in Brazil, Malaysia, or China, whose objectives may be far different from those assumed in this paper. Rather the goal of our theory is to indicate what a government with the ability to manipulate interest rates and other prices should do (at least from a unilateral perspective). Of course, this does not

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[18]In our simulations we have also encountered the case in which the largest of the two countries, Foreign, wins the capital control war. For instance, if Home starts with 1/12 rather than 1/6 of world endowments, while still ending up with 1/3 in the long-run, Foreign consumption is higher by 0.08% in the Nash equilibrium compared to the non capital controls benchmark. This resonates well with existing results in the trade literature indicating that large countries sometimes win trade wars; see e.g. Johnson (1953-54), Kennan and Riezman (1988), and Syropoulos (2002).

imply that our theory has no practical implications for actual policy coordination efforts. In the trade literature, optimal tariff arguments upon which the present analysis builds have paved the way for a rich positive theory of international trade agreements; see Bagwell and Staiger (2002). We hope that our analysis will provide a useful starting point for thinking about agreements on capital controls as well as other related questions at the frontier of international macro and international trade policy.

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7 Appendix

Proof of Proposition 1. The first-order condition associated with Home's planning problem implies:

$$u'(c_t) - \mu \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right] = 0. \tag{33}$$

Differentiating equation (33), we get after simple rearrangements:

$$\frac{\partial c_t}{\partial y_t} = \frac{\mu u^{*\prime\prime}(Y - c_t)}{u''(c_t) - \mu \frac{\partial}{\partial c_t} \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right]} > 0, \tag{34}$$

where the inequality directly derives from the strict concavity of u and u^* and the strict convexity of $u^{*\prime}(Y - c_t)(c_t - y_t)$. Inequality (34) implies that for any pair of periods, t and s, such that $y_s > y_t$, we must have $c_s > c_t$. ∎

Section 3.4. Let us focus on date 0 and date 1. Let $\{c_t\}_{t=1}^{\infty}$ and $\{c_t'\}_{t=1}^{\infty}$ denote the optimal consumption paths for all dates $t \geq 1$ from the point of view of the home government at date 0 and date 1, respectively. We want show that one can construct $\{a_{1,s}^*\}_{s=1}^{\infty}$ satisfying equation (14) at $t = 0$ such that $c_t' = c_t$ for all $t \geq 1$. As in Lucas and Stokey (1983), we focus on the first-order conditions associated with Home's planning problem at dates $t = 0$ and $t = 1$. In the present environment, they imply

$$u'(c_t) = \mu_0 \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t + a_{0,t}^*) \right], \tag{35}$$

$$u'(c_t') = \mu_1 \left[u^{*\prime}(Y - c_t') - u^{*\prime\prime}(Y - c_t')(c_t' - y_t + a_{1,t}^*) \right], \tag{36}$$

where μ_0 and μ_1 are the Lagrange multipliers associated with the implementability constraints of the home government at dates 0 and 1, respectively. For a given value of μ_1, let us construct $a_{1,t}^*(\mu_1)$ such that

$$\mu_0 \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t + a_{0,t}^*) \right]$$
$$= \mu_1 \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t + a_{1,t}^*(\mu_1)) \right],$$

which can be rearranged as

$$a_{1,t}^*(\mu_1) = \frac{u^{*\prime}(Y - c_t)}{u^{*\prime\prime}(Y - c_t)} - (c_t - y_t) + \frac{\mu_0}{\mu_1} \left[a_{0,t}^* - \frac{u^{*\prime}(Y - c_t)}{u^{*\prime\prime}(Y - c_t)} + (c_t - y_t) \right]. \tag{37}$$

By construction, if the previous condition holds, then, for any μ_1, equation (36) holds as

well if $c'_t = c_t$ for all $t \geq 1$. Now let us choose μ_1 such that:

$$\mu_1 = \mu_0 \sum_{s=1}^{\infty} \nu_s \left[1 - \frac{u^{*\prime\prime}(Y - c_s)\left(c_s - y_s + a^*_{0,s}\right)}{u^{*\prime}(Y - c_s)} \right], \tag{38}$$

where

$$\nu_s \equiv \frac{[u^{*\prime}(Y - c_s)]^2 / u^{*\prime\prime}(Y - c_s)}{\sum_{r=1}^{\infty} [u^{*\prime}(Y - c_r)]^2 / u^{*\prime\prime}(Y - c_r)} \in [0, 1]. \tag{39}$$

By equation (35), we know that $1 - \frac{u^{*\prime\prime}(Y-c_s)(c_s-y_s+a^*_{0,s})}{u^{*\prime}(Y-c_s)} > 0$. Thus we have $\mu_1 > 0$. One can check that, by construction, equations (37)-(39) further imply

$$\sum_{s=1}^{\infty} u^{*\prime}(Y - c_s) \left[c_s - y_s + a^*_{1,s}(\mu_1) \right] = 0.$$

Thus, equation (14) is satisfied at $t = 0$. Since equations (14) and (15) evaluated at $t = 0$ and $t = 1$, respectively, are identical, we have constructed $\{a^*_{1,s}\}_{s=1}^{\infty}$ satisfying equation (14) at $t = 0$ such that $c'_t = c_t$ for all $t \geq 1$. The argument for other dates is similar. ∎

Proof of Proposition 3. The basic strategy is the same as in the proof of Proposition 1. The first-order condition associated with Home's planning problem implies:

$$U'(C_t) - \mu \left(\sum_i \rho_i(C_t) \frac{\partial c_i(C_t)}{\partial C_t} + \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t}(c_i(C_t) - y_{it}) \right) = 0, \tag{40}$$

Differentiating equation (40), we get after simple rearrangements

$$dC_t = -\frac{\mu \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it}}{U''(C_t) - \mu \frac{\partial}{\partial C_t}\left[\sum_i \rho_i(C_t)\frac{\partial c_i(C_t)}{\partial C_t} - \sum_i \frac{\partial \rho_i(C_t)/\partial C_t}{\rho_i(C_t)}\rho_i(C_t)(c_i(C_t) - y_{it}) \right]}.$$

By the strict concavity of U and the strict convexity of $\rho(C_t) \cdot (c(C_t) - y_t)$, we therefore have $dC_t > 0$ if and only if $\sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0$. ∎

Proof of Proposition 4. In the main text, we have already established that

$$\theta_t = 1 - \left(\frac{1 + \tau_t}{1 + \tau_{t+1}} \right) \left(\frac{P_{t+1}/P^*_{t+1}}{P_t/P^*_t} \right), \tag{41}$$

with the wedge τ_t such that

$$\tau_t = \frac{U'(C_t)}{\mu U^{*\prime}(C^*(C_t))} - 1.$$

Since U and U^* are concave and C^* is decreasing in C_t along the Pareto frontier, we al-

ready know from Proposition 3 that

$$\tau_{t+1} < \tau_t \text{ if and only if } \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0. \tag{42}$$

Now notice that by the envelope theorem, $C^{*\prime}(C_t)$ is equal to the opposite of the Lagrange multiplier associated with the constraint $g(c) \geq C_t$ in (18). Thus the first-order conditions associated with that program imply

$$g_i^*(c_t) = -C^{*\prime}(C_t) g_i(c_t). \tag{43}$$

Let us now show that $P_t = p_{it}/g_i(c_t)$. Let us denote $c_t(1) \equiv \arg\min_c \{p_t \cdot c : g(c) \geq 1\}$. The associated first-order conditions are given by (i) $p_i = \lambda g_i[c_t(1)]$ and (ii) $g[c_t(1)] = 1$. This implies

$$P_t = \sum_i p_{it} c_{it}(1) = \lambda \sum_i g_i[c_t(1)] c_{it}(1) = \lambda g[c_t(1)] = \lambda,$$

where the third equality uses the fact that g is homogeneous of degree one. Combining this equality with condition (i), we obtain $P_t = p_{it}/g_i[c_t(1)]$. Since g_i is homogeneous of degree zero, this further implies $P_t = p_{it}/g_i(c_t)$ for all $i = 1, ..., n$. The same logic applied to Foreign implies $P_t^* = p_{it}/g_i^*(c_t)$. Combining the two previous observations with equation (43), we obtain

$$\frac{P_t}{P_t^*} = -C^{*\prime}(C_t).$$

Since g and g^* are concave and homogeneous of degree one, standard arguments imply that the solution C^* of (18) is (weakly) concave in C_t. By Proposition 3, we therefore have

$$P_{t+1}/P_{t+1}^* > P_t/P_t^* \text{ if and only if } \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0. \tag{44}$$

Combining equation (41) with conditions (42) and (44), we finally get $\theta_t < 0$ if and only if $\sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0.$ ∎

Proof of Proposition 5. Suppose that $dy_{it+1} > 0$ and $dy_{jt+1} = 0$. By Proposition 4, we know that $\theta_t < 0$ if and only if $\partial \rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$, where $\rho(C_t) \equiv U^{*\prime}(C^*(C_t)) \nabla g^*(c^*(C_t))$. Thus if $dy_{it+1} > 0$ and $dy_{jt+1} = 0$, $\theta_t < 0$ if and only if

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))} \frac{\partial C^*(C_t)}{\partial C_t} + \left[\frac{g_{ii}^*(c^*(C_t))}{g_i^*(c^*(C_t))} \frac{\partial c_i^*(C_t)}{\partial C_t} + \frac{g_{ij}^*(c^*(C_t))}{g_i^*(c^*(C_t))} \frac{\partial c_j^*(C_t)}{\partial C_t} \right] > 0. \tag{45}$$

Consider the first term on the left-hand side of inequality (45). In the proof of Proposition 4, we have already established that

$$\frac{\partial C^*(C_t)}{\partial C_t} = -\frac{P_t}{P_t^*}.$$

Since $U^*(C^*) = \frac{1}{1-\gamma}(C^*)^{1-\gamma}$, we therefore have

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))}\frac{\partial C^*(C_t)}{\partial C_t} = \frac{\gamma P_t}{P_t^* C_t^*}. \qquad (46)$$

Let us now turn to the second term on the left-hand side of inequality (45). Our goal is to establish that

$$\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} = \frac{\alpha}{(1-\alpha)C_t}\frac{(2\alpha-1)\,c_2\,(C_t)}{(1-\alpha)\,c_2^*\,(C_t) + \alpha c_2\,(C_t)}, \qquad (47)$$

$$\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} = -\frac{1}{C_t}\frac{(2\alpha-1)\,c_2\,(C_t)}{(1-\alpha)\,c_2^*\,(C_t) + \alpha c_2\,(C_t)}. \qquad (48)$$

Since $g^*(c^*) = (c_2^*)^\alpha\,(c_1^*)^{1-\alpha}$, simple algebra implies

$$\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)} = -\frac{\alpha}{c_{1t}^*},\quad \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)} = \frac{\alpha}{c_{2t}^*},$$

$$\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)} = \frac{(1-\alpha)}{c_{1t}^*},\quad \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)} = -\frac{(1-\alpha)}{c_{2t}^*}.$$

Using the previous expressions, we obtain

$$\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} = \alpha\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}, \qquad (49)$$

$$\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} = -(1-\alpha)\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}. \qquad (50)$$

Let us compute $\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}$. By definition, $c\,(C_t)$ and $c^*\,(C_t)$ are the solution of

$$\max_{c,c^*}\ (c_2^*)^\alpha\,(c_1^*)^{1-\alpha}$$

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subject to

$$c_1 + c_1^* \leq Y_1,$$

$$c_2 + c_2^* \leq Y_2,$$

$$c_1^\alpha c_2^{1-\alpha} \geq C_t.$$

The associated first-order conditions imply

$$\frac{c_2^* \, (C_t)}{c_1^* \, (C_t)} = \beta \left(\frac{Y_2 - c_2^* \, (C_t)}{Y_1 - c_1^* \, (C_t)} \right) \text{ with } \beta \equiv [\alpha / (1 - \alpha)]^2, \tag{51}$$

and

$$\frac{Y_2 - c_2^* \, (C_t)}{Y_1 - c_1^* \, (C_t)} = \left(\frac{C_t}{Y_2 - c_2^* \, (C_t)} \right)^{-\frac{1}{\alpha}}. \tag{52}$$

Combining the two previous expressions, we obtain

$$\frac{\partial \ln \left[c_2^*(C_t) / c_1^*(C_t) \right]}{\partial C_t} = -\frac{1}{\alpha} \left(\frac{1}{C_t} - \frac{\partial \ln \left[Y_2 - c_2^*(C_t) \right]}{\partial C_t} \right) = -\frac{1}{\alpha} \left(\frac{1}{C_t} - \frac{\partial \ln \left[c_2(C_t) \right]}{\partial C_t} \right). \tag{53}$$

Let us compute $\frac{\partial \ln[c_2(C_t)]}{\partial C_t}$. Using the resource constraint, we can express equation (51) as

$$c_1 \, (C_t) = \frac{\beta c_2 \, (C_t) \, Y_1}{Y_2 - (1 - \beta) \, c_2 \, (C_t)}.$$

Together with equation (52), using again the resource constraint, this implies

$$c_2 \, (C_t) = C_t \left[\frac{Y_2 - (1 - \beta) \, c_2 \, (C_t)}{\beta Y_1} \right]^\alpha.$$

Taking the log and differentiating, we obtain after rearrangements

$$\frac{\partial \ln \left[c_2(C_t) \right]}{\partial C_t} = \frac{Y_2 - (1 - \beta) \, c_2 \, (C_t)}{C_t \left[Y_2 - (1 - \alpha) \, (1 - \beta) \, c_2 \, (C_t) \right]}. \tag{54}$$

Equations (53) and (54) imply

$$\frac{\partial \ln \left[c_2^*(C_t) / c_1^*(C_t) \right]}{\partial C_t} = \frac{1}{C_t} \frac{(\beta - 1) \, c_2 \, (C_t)}{Y_2 + (1 - \alpha) \, (\beta - 1) \, c_2 \, (C_t)}.$$

Using the definition of $\beta \equiv [\alpha/(1-\alpha)]^2$, we can rearrange the previous expression as

$$\frac{\partial \ln [c_2^*(C_t)/c_1^*(C_t)]}{\partial C_t} = \frac{1}{(1-\alpha)C_t} \frac{(2\alpha - 1) c_2 (C_t)}{(1-\alpha) c_2^* (C_t) + \alpha c_2 (C_t)}.$$

Equations (47) and (48) directly derive from the previous expression and equations (49) and (50), respectively.

To conclude the proof of Proposition 5, first note that equations (46) and (47) imply

$$\frac{U^{*\prime\prime} (C^*(C_t))}{U^{*\prime} (C^*(C_t))} \frac{\partial C^*(C_t)}{\partial C_t} + \left[\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)} \frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)} \frac{\partial c_2^*(C_t)}{\partial C_t} \right]$$
$$= \frac{\gamma P_t}{P_t^* C_t^*} + \frac{\alpha}{(1-\alpha)C_t} \frac{(2\alpha - 1) c_2 (C_t)}{(1-\alpha) c_2^* (C_t) + \alpha c_2 (C_t)} > 0.$$

Thus if $dy_{1t+1} > 0$ and $dy_{2t+1} = 0$, then $\theta_t < 0$. Second note that equations (46) and (48) imply

$$\frac{U^{*\prime\prime} (C^*(C_t))}{U^{*\prime} (C^*(C_t))} \frac{\partial C^*(C_t)}{\partial C_t} + \left[\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)} \frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)} \frac{\partial c_2^*(C_t)}{\partial C_t} \right]$$
$$= \frac{\gamma P_t}{P_t^* C_t^*} - \frac{1}{C_t} \frac{(2\alpha - 1) c_2 (C_t)}{(1-\alpha) c_2^* (C_t) + \alpha c_2 (C_t)}.$$

According to this expression, if $dy_{1t+1} = 0$ and $dy_{2t+1} > 0$, then $\theta_t < 0$ if and only if

$$\gamma > \frac{P_t^* C_t^*}{P_t C_t} \frac{(2\alpha - 1) c_2 (C_t)}{(1-\alpha) c_2^* (C_t) + \alpha c_2 (C_t)}. \tag{55}$$

Since utility functions are Cobb-Douglas, $g(c) = c_1^\alpha c_2^{1-\alpha}$ and $g^*(c^*) = (c_2^*)^\alpha (c_1^*)^{1-\alpha}$, we know that

$$p_{2t}c_2 (C_t) = (1-\alpha) P_t C_t$$
$$p_{2t}c_2^* (C_t) = \alpha P_t^* C_t^*$$

Combining these two observations with inequality (55), we conclude that if $dy_{1t+1} = 0$ and $dy_{2t+1} > 0$, then $\theta_t < 0$ if and only if $\gamma > \left(\frac{2\alpha - 1}{\alpha}\right) \left(\frac{P_t^* C_t^*}{P_t^* C_t^* + P_t C_t}\right)$. ∎

Section 4.5. Consider first the Pareto Problem. By definition, $c (C_t)$ and $c^* (C_t)$ solve

$$\max_{c,c^*} c_1^{*1/N} c_2^{*1-1/N}$$

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subject to

$$c_1 + c_1^* \leq Y_1,$$

$$c_2 + c_2^* \leq Y_2 N,$$

$$c_1^{1/2} c_2^{1/2} \geq C_t.$$

The associated first-order conditions imply

$$\frac{c_2^*}{c_1^*} = (N-1)\frac{(NY_2 - c_2^*)}{(Y_1 - c_1^*)},$$

$$(Y_1 - c_1^*)^{1/2} (NY_2 - c_2^*)^{1/2} = C_t.$$

Combining these two expressions we get

$$(c_2^*)^2 + \left[\frac{(N-2)C_t^2}{(N-1)Y_1} - 2NY_2 \right] c_2^* + \left[(NY_2)^2 - N\left(\frac{Y_2}{Y_1}\right)C_t^2 \right] = 0.$$

This implies

$$c_2^* (C_t) = NY_2 - \frac{1}{2}\left\{ \frac{(N-2)C_t^2}{(N-1)Y_1} + \sqrt{\left[\frac{(N-2)C_t^2}{(N-1)Y_1}\right]^2 + 4\left(\frac{N}{N-1}\right)\left(\frac{Y_2}{Y_1}\right)C_t^2} \right\}.$$

In turn, we obtain

$$c_1^* (C_t) = \frac{NY_2 - \frac{1}{2}\left\{ \frac{(N-2)C_t^2}{(N-1)Y_1} + \sqrt{\left[\frac{(N-2)C_t^2}{(N-1)Y_1}\right]^2 + 4\left(\frac{N}{N-1}\right)\left(\frac{Y_2}{Y_1}\right)C_t^2} \right\}}{NY_2 + \frac{(N-2)}{2}\left\{ \frac{(N-2)C_t^2}{(N-1)Y_1} + \sqrt{\left[\frac{(N-2)C_t^2}{(N-1)Y_1}\right]^2 + 4\left(\frac{N}{N-1}\right)\left(\frac{Y_2}{Y_1}\right)C_t^2} \right\}}.$$

Using the two previous expressions, we can compute

$$C^* (C_t) =$$

$$\left\{ \left(\frac{N}{N-1}\right)Y_2 - \frac{1}{2(N-1)}\left\{ \frac{(N-2)C_t^2}{(N-1)Y_1} + \sqrt{\left[\frac{(N-2)C_t^2}{(N-1)Y_1}\right]^2 + 4\left(\frac{N}{N-1}\right)\left(\frac{Y_2}{Y_1}\right)C_t^2} \right\} \right\}$$

$$\times \left\{ NY_2 + \frac{(N-2)}{2}\left\{ \frac{(N-2)C_t^2}{(N-1)Y_1} + \sqrt{\left[\frac{(N-2)C_t^2}{(N-1)Y_1}\right]^2 + 4\left(\frac{N}{N-1}\right)\left(\frac{Y_2}{Y_1}\right)C_t^2} \right\} \right\}^{-1/N},$$

$$(N-1) g_1^*(c^* (C_t)) =$$

$$\frac{1}{N} \left(NY_2 + \frac{(N-2)}{2} \left\{ \frac{(N-2)C_t^2}{(N-1)Y_1} + \sqrt{\left[\frac{(N-2)C_t^2}{(N-1)Y_1} \right]^2 + 4 \left(\frac{N}{N-1} \right) \left(\frac{Y_2}{Y_1} \right) C_t^2} \right\} \right)^{1-1/N},$$

and

$$(N-1) g_2^*(c^* (C_t)) =$$

$$\left(1 - \frac{1}{N} \right) \left\{ NY_2 + \frac{(N-2)}{2} \left\{ \frac{(N-2)C_t^2}{(N-1)Y_1} + \frac{1}{2}\sqrt{\left[\frac{(N-2)C_t^2}{(N-1)Y_1} \right]^2 + 4 \left(\frac{N}{N-1} \right) \left(\frac{Y_2}{Y_1} \right) C_t^2} \right\} \right\}^{-1/N}.$$

As N goes to infinity, we therefore get

$$\lim_{N \to \infty} C^* (C_t) = Y_2,$$

$$\lim_{N \to \infty} (N-1) g_1^*(c^* (C_t)) = Y_2 + \frac{1}{2} \left\{ \frac{C_t^2}{Y_1} + \sqrt{\left(\frac{C_t^2}{Y_1} \right)^2 + 4 \left(\frac{Y_2}{Y_1} \right) C_t^2} \right\},$$

$$\lim_{N \to \infty} (N-1) g_2^*(c^* (C_t)) = 1.$$

As N goes to infinity the constraint (19) associated with Home's planning problem therefore converges towards

$$\sum_{t=0}^{\infty} \beta^t Y_2^{-\gamma} \left[\frac{g_1^*(c^* (C_t))}{g_2^*(c^* (C_t))} (c_1 (C_t) - y_{1t}) + (c_2 (C_t) - y_{2t}) \right] = 0,$$

where $g_1^*(c^* (C_t))/g_2^*(c^* (C_t)) = Y_2 + \frac{1}{2} \left[C_t^2/Y_1 + \sqrt{(C_t^2/Y_1)^2 + 4Y_2 (C_t^2/Y_1)} \right].$ ∎

Proof of Lemma 1. A Nash equilibrium corresponds to a pair of sequences of taxes on one-period bonds, $(\theta \equiv \{\theta_t\}, \theta^* \equiv \{\theta_t^*\})$, such that

$$\theta \in \arg\max_{\tilde{\theta}} \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t (\tilde{\theta}, \theta^*)) | \sum_{t=0}^{\infty} p_t (\tilde{\theta}, \theta^*) (c_t (\tilde{\theta}, \theta^*) - y_t) = 0 \right\}, \qquad (56)$$

$$\theta^* \in \arg\max_{\tilde{\theta}^*} \left\{ \sum_{t=0}^{\infty} \beta^t u^*(c_t^* (\theta, \tilde{\theta}^*)) | \sum_{t=0}^{\infty} p_t (\theta, \tilde{\theta}^*) (c_t^* (\theta, \tilde{\theta}^*) - y_t^*) = 0 \right\}, \qquad (57)$$

where the pair of consumption sequences, $(c (\theta, \theta^*) \equiv \{c_t (\theta, \theta^*)\}, c^* (\theta, \theta^*) \equiv \{c_t^* (\theta, \theta^*)\})$, and the sequence of prices, $p (\theta, \theta^*) \equiv \{p_t (\theta, \theta^*)\}$, are such that consumers maximize

their utility in both countries,

$$c\left(\theta, \theta^*\right) \in \arg\max_c \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) | \sum_{t=0}^{\infty} \frac{p_t\left(\theta, \theta^*\right)}{\prod_{s=0}^{t-1}\left(1-\theta_s\right)}(c_t - y_t) = L\left(\theta, \theta^*\right) \right\}, \qquad (58)$$

$$c^*\left(\theta, \theta^*\right) \in \arg\max_{c^*} \left\{ \sum_{t=0}^{\infty} \beta^t u^*(c_t^*) | \sum_{t=0}^{\infty} \frac{p_t\left(\theta, \theta^*\right)}{\prod_{s=0}^{t-1}\left(1-\theta_s^*\right)}(c_t^* - y_t^*) = L^*\left(\theta, \theta^*\right) \right\}, \qquad (59)$$

and markets clear in every period,

$$c_t\left(\theta, \theta^*\right) + c_t^*\left(\theta, \theta^*\right) = Y, \qquad (60)$$

with $L\left(\theta, \theta^*\right)$ and $L^*\left(\theta, \theta^*\right)$ the total tax revenues in Home and Foreign, respectively.

In this proof, we will first focus on the following primal problems:

$$c \in \arg\max_{\tilde{c}} \left\{ \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t) | \sum_{t=0}^{\infty} \beta^t u^{*\prime}\left(Y - \tilde{c}_t\right) \left[\prod_{s=0}^{t-1}\left(1-\theta_s^*\right)\right] (\tilde{c}_t - y_t) = 0 \right\}, \qquad (61)$$

$$c^* \in \arg\max_{\tilde{c}^*} \left\{ \sum_{t=0}^{\infty} \beta^t u^*(\tilde{c}_t^*) | \sum_{t=0}^{\infty} \beta^t u^{\prime}\left(Y - \tilde{c}_t^*\right) \left[\prod_{s=0}^{t-1}\left(1-\theta_s\right)\right] (\tilde{c}_t^* - y_t^*) = 0 \right\}, \qquad (62)$$

for some guessed sequences of taxes, (θ, θ^*), and show that there exists consumption sequences, (c, c^*), that solves the primal problems, (61) and (62). We will then verify that if (c, c^*) solves the primal problems, (61) and (62), then (θ, θ^*) is a Nash equilibrium that solves (56) and (57). We proceed in four steps.

Step 1: *For any $\alpha > 0$ and $y \in (0, Y)$, there exists a unique $c(\alpha, y)$ such that*

$$\frac{u'(c) + u''(c)(c - y)}{u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y)} = \alpha. \qquad (63)$$

Furthermore $c(\alpha, y)$ is continuous, strictly decreasing in α, and strictly increasing in y.

To see this, first note that Conditions (i) and (ii) imply that $u'(c) + u''(c)(c - y) > 0$, $u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y) > 0$, $u'(c) + u''(c)(c - y)$ is continuous and strictly decreasing in c, and $u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y)$ is continuous and strictly increasing in c. This further implies that $F(c, y) \equiv \frac{u'(c) + u''(c)(c-y)}{u^{*\prime}(Y-c) - u^{*\prime\prime}(Y-c)(c-y)}$ is continuous and strictly decreasing in c. From Conditions (i) and (iii), we also know that

$$\lim_{c \to 0} u'(c) + u''(c)(c - y) \geq \lim_{c \to 0} u'(c) = \infty,$$
$$\lim_{c \to Y} u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y) \geq \lim_{c^* \to 0} u^{*\prime}(c^*) = \infty,$$

which implies $\lim_{c\to 0} F(c,y) = \infty$ and $\lim_{c\to Y} F(c,y) = 0$. By the Intermediate Value Theorem, for any $\alpha > 0$, there therefore exists $c(\alpha, y)$ such that equation (63) holds. Furthermore since F is strictly decreasing in c, $c(\alpha, y)$ is unique, continuous, and strictly decreasing in α. Finally since $F(c,y)$ is continuous and strictly increasing in y, by the strict concavity of u and u^*, $c(\alpha, y)$ must be continuous and strictly increasing in y.

Step 2: *For any sequence $\{y_t\}$, there exists $\alpha^0 > 0$ such that*

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}\left(Y - c\left(\alpha^0, y_t\right)\right)\left[\prod_{s=0}^{t-1}\left(1 - \theta_s^*\left(\alpha^0\right)\right)\right]\left(c\left(\alpha^0, y_t\right) - y_t\right) = 0, \qquad (64)$$

$$\sum_{t=0}^{\infty} \beta^t u'\left(c\left(\alpha^0, y_t\right)\right)\left[\prod_{s=0}^{t-1}\left(1 - \theta_s\left(\alpha^0\right)\right)\right]\left(y_t - c\left(\alpha^0, y_t\right)\right) = 0, \qquad (65)$$

with the sequences $\{\theta_t(\alpha)\}$ and $\{\theta_t^*(\alpha)\}$ constructed such that

$$1 - \theta_t(\alpha) = \frac{1 - \frac{u^{*\prime\prime}(Y - c(\alpha, y_t))}{u^{*\prime}(Y - c(\alpha, y_t))}\left(c(\alpha, y_t) - y_t\right)}{1 - \frac{u^{*\prime\prime}(Y - c(\alpha, y_{t+1}))}{u^{*\prime}(Y - c(\alpha, y_{t+1}))}\left(c(\alpha, y_{t+1}) - y_{t+1}\right)}, \qquad (66)$$

$$1 - \theta_t^*(\alpha) = \frac{1 - \frac{u''(c(\alpha, y_t))}{u'(c(\alpha, y_t))}\left(y_t - c(\alpha, y_t)\right)}{1 - \frac{u''(c(\alpha, y_{t+1}))}{u'(c(\alpha, y_{t+1}))}\left(y_{t+1} - c(\alpha, y_{t+1})\right)}. \qquad (67)$$

Using equation (67), we can rearrange equation (64) as

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}\left(Y - c(\alpha, y_t)\right) \frac{u^{*\prime}\left(Y - c(\alpha, y_t)\right)\left(c(\alpha, y_t) - y_t\right)}{u^{*\prime}\left(Y - c(\alpha, y_t)\right) - u^{*\prime\prime}\left(Y - c(\alpha, y_t)\right)\left(c(\alpha, y_t) - y_t\right)} = 0.$$

Let us denote

$$g(\alpha, y_t) \equiv u^{*\prime}\left(Y - c(\alpha, y_t)\right) \frac{u^{*\prime}\left(Y - c(\alpha, y_t)\right)\left(c(\alpha, y_t) - y_t\right)}{u^{*\prime}\left(Y - c(\alpha, y_t)\right) - u^{*\prime\prime}\left(Y - c(\alpha, y_t)\right)\left(c(\alpha, y_t) - y_t\right)}.$$

We know that $u^{*\prime}(Y - c)$, $u^{*\prime}(Y - c)(c - y_t)$, and $u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y_t) > 0$ are continuous functions of c. Thus $u^{*\prime}(Y - c)\frac{u^{*\prime}(Y - c)(c - y_t)}{u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y_t)}$ is continuous in c. Since $c(\alpha, y_t)$ is continuous in α for all y_t by Step 1, $g(\alpha, y_t)$ is continuous in α. Similarly, $u^{*\prime}(Y - c)(c - y_t)$ and $u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y_t) > 0$ are continuous functions of y_t. Thus $u^{*\prime}(Y - c)\frac{u^{*\prime}(Y - c)(c - y_t)}{u^{*\prime}(Y - c) - u^{*\prime\prime}(Y - c)(c - y_t)}$ is continuous in y_t. Since $c(\alpha, y_t)$ is continuous in y_t for all α by Step 1, $g(\alpha, y_t)$ is continuous in y_t.

By Condition (iii), there exists $\varepsilon > 0$ such that $y_t \in [\varepsilon, Y - \varepsilon]$ for all t. By equation (63) and the boundary condition $\lim_{c^* \to 0} u^{*\prime}(c^*) = \infty$, we know that, for any $y \in [\varepsilon, Y - \varepsilon]$, $\lim_{\alpha \to 0} c(\alpha, y) = Y$. Thus there must exist $\underline{\alpha} > 0$ such that $c(\underline{\alpha}, \varepsilon) > Y - \varepsilon$. By equa-

tion (63) and the boundary condition $\lim_{c \to 0} u'(c) = \infty$, we also know that, for any $y \in [\varepsilon, Y - \varepsilon]$, $\lim_{\alpha \to \infty} c(\alpha, y) = 0$. Thus there must exist $\bar{\alpha} > 0$ such that $c(\bar{\alpha}, Y - \varepsilon) < \varepsilon$. Since $c(\alpha, y)$ is increasing in y by Step 1, we must therefore have $c(\underline{\alpha}, y) > Y - \varepsilon$ and $c(\bar{\alpha}, y) < \varepsilon$ for all $y \in [\varepsilon, Y - \varepsilon]$. Furthermore since $c(\alpha, y)$ is decreasing in α by Step 1 and $c(\bar{\alpha}, \varepsilon) < c(\bar{\alpha}, Y - \varepsilon) < \varepsilon < Y - \varepsilon < c(\underline{\alpha}, \varepsilon)$, we must have $\bar{\alpha} > \underline{\alpha}$.

Let us now restrict ourselves to $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Since $g(\alpha, y_t)$ is continuous in (α, y_t), there must exist $M > 0$ such that $|g(\alpha, y_t)| \leq M$ for all $(\alpha, y) \in [\underline{\alpha}, \bar{\alpha}] \times [\varepsilon, Y - \varepsilon]$. We therefore have $|\beta^t g(\alpha, y_t)| \leq M\beta^t$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ and all t. By Weierstrass criterion, $G_T(\alpha) \equiv \sum_{t=0}^{T} \beta^t g(\alpha, y_t)$ therefore converges uniformly towards $G_\infty(\alpha) \equiv \sum_{t=0}^{\infty} \beta^t g(\alpha, y_t)$. And by the uniform convergence theorem and the continuity of $\beta^t g(\alpha, y_t)$ in α for all t, $G_\infty(\alpha)$ must be continuous in α. By construction of $\underline{\alpha}$ and $\bar{\alpha}$, we have $\beta^t g(\underline{\alpha}, y_t) > 0$ for all t and $\beta^t g(\bar{\alpha}, y_t) < 0$ for all t. Thus $G_\infty(\underline{\alpha}) > 0$ and $G_\infty(\bar{\alpha}) < 0$. Since $G_\infty(\alpha)$ is continuous in α, the Intermediate Value Theorem implies the existence of α^0 such that equation (64) holds.

To conclude one can use equations (63), (66), and (67) to check that if equation (64) holds, then equation (65) holds as well.

Step 3: *For any sequence* $\{y_t\}$, $\{c(\alpha^0, y_t)\}$ *and* $\{Y - c(\alpha^0, y_t)\}$ *are a solution of the primal problems,* (61) *and* (62), *for* $\{\theta_s\} \equiv \{\theta_s(\alpha^0)\}$ *and* $\{\theta_s^*\} \equiv \{\theta_s^*(\alpha^0)\}$.

Let us first show that there exists $\mu > 0$ such that for all t,

$$u'(c(\alpha^0, y_t)) = \mu \left[\prod_{s=0}^{t-1} \left(1 - \theta_s^*(\alpha^0)\right) \right] \left[u^{*\prime}(Y - c(\alpha^0, y_t)) - u^{*\prime\prime}(Y - c(\alpha^0, y_t))(c(\alpha^0, y_t) - y_t) \right]$$
(68)

By construction of $\{\theta^*(\alpha)\}$, we know that for any α:

$$1 - \theta_t^*(\alpha) = \frac{1 - \frac{u''(c(\alpha, y_t))}{u'(c(\alpha, y_t))}(y_t - c(\alpha, y_t))}{1 - \frac{u''(c(\alpha, y_{t+1}))}{u'(c(\alpha, y_{t+1}))}(y_{t+1} - c(\alpha, y_{t+1}))}.$$

Thus

$$\left[\prod_{s=0}^{t-1} \left(1 - \theta_s^*(\alpha^0)\right) \right] = \frac{1 - \frac{u''(c(\alpha^0, y_0))}{u'(c(\alpha^0, y_0))}(y_0 - c(\alpha^0, y_0))}{1 - \frac{u''(c(\alpha^0, y_t))}{u'(c(\alpha^0, y_t))}(y_t - c(\alpha^0, y_t))}.$$

and in turn,

$$\left[\prod_{s=0}^{t-1} \left(1 - \theta_s^*(\alpha^0)\right) \right] \frac{[u^{*\prime}(c_t^*) - u^{*\prime\prime}(c_t^*)(c_t - y_t)]}{u'(c_t)}$$

$$= \left[1 - \frac{u''(c(\alpha^0, y_0))}{u'(c(\alpha^0, y_0))}(y_0 - c(\alpha^0, y_0)) \right] \frac{u^{*\prime}(Y - c(\alpha^0, y_t)) - u^{*\prime\prime}(Y - c(\alpha^0, y_t))(c(\alpha^0, y_t) - y_t)}{u'(c(\alpha^0, y_t)) - u''(c(\alpha^0, y_t))(y_t - c(\alpha^0, y_t))}.$$

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By equation (63), we therefore have

$$\left[\prod_{s=0}^{t-1}\left(1-\theta_s^*\left(\alpha^0\right)\right)\right]\frac{\left[u^{*\prime}(c_t^*)-u^{*\prime\prime}(c_t^*)(c_t-y_t)\right]}{u'(c_t)}=\frac{\left[1-\frac{u''\left(c\left(\alpha^0,y_0\right)\right)}{u'\left(c\left(\alpha^0,y_0\right)\right)}\left(y_0-c\left(\alpha^0,y_0\right)\right)\right]}{\alpha^0}.$$

This implies

$$u'(c_t)=\mu\left[\prod_{s=0}^{t-1}\left(1-\theta_s^*\left(\alpha^0\right)\right)\right]\left[u^{*\prime}(c_t^*)-u^{*\prime\prime}(c_t^*)(c_t-y_t)\right],$$

with $\mu\equiv\alpha^0/\left[1-\frac{u''\left(c\left(\alpha^0,y_0\right)\right)}{u'\left(c\left(\alpha^0,y_0\right)\right)}\left(y_0-c\left(\alpha^0,y_0\right)\right)\right]$. By definition of $c\left(\alpha^0,y_0\right)$, we know that

$$\alpha^0=\frac{u'\left(c\left(\alpha^0,y_0\right)\right)+u''\left(c\left(\alpha^0,y_0\right)\right)\left(c\left(\alpha^0,y_0\right)-y_0\right)}{u^{*\prime}\left(Y-c\left(\alpha^0,y_0\right)\right)-u^{*\prime\prime}\left(Y-c\left(\alpha^0,y_0\right)\right)\left(c\left(\alpha^0,y_0\right)-y_0\right)}.$$

Thus, after simplifications, we get:

$$\mu=\frac{u'\left(c\left(\alpha^0,y_0\right)\right)}{u^{*\prime}\left(Y-c\left(\alpha^0,y_0\right)\right)-u^{*\prime\prime}\left(Y-c\left(\alpha^0,y_0\right)\right)\left(c\left(\alpha^0,y_0\right)-y_0\right)}.$$

Since $u'\left(c\left(\alpha^0,y_0\right)\right)>0$ and $u^{*\prime}\left(Y-c\left(\alpha^0,y_0\right)\right)-u^{*\prime\prime}\left(Y-c\left(\alpha^0,y_0\right)\right)\left(c\left(\alpha^0,y_0\right)-y_0\right)>0$, we have established equation (68). By construction of α^0, we also know that equation (64) holds. Thus we have constructed a sequence $\{c(\alpha^0,y)\}$ that satisfies the first-order conditions associated with (61) for $\{\theta_s^*\}\equiv\{\theta_s^*\left(\alpha^0\right)\}$. Since $u\left(c\right)$ and $u^{*\prime}\left(Y-c\right)\left(c-y\right)$ are strictly concave functions of c, these first-order conditions are also sufficient. Thus $\{c\left(\alpha^0,y_t\right)\}$ is a solution of (61) for $\{\theta_s^*\}\equiv\{\theta_s^*\left(\alpha^0\right)\}$. Using the exact same logic, one can show that $\{Y-c(\alpha^0,y)\}$ is a solution of (62) for $\{\theta_s\}\equiv\{\theta_s\left(\alpha^0\right)\}$.

Step 4: *For any sequence $\{y_t\}$, if $\{c\left(\alpha^0,y_t\right)\}$ and $\{Y-c\left(\alpha^0,y_t\right)\}$ are a solution of the primal problems, (61) and (62), for $\{\theta_s\}\equiv\{\theta_s\left(\alpha^0\right)\}$ and $\{\theta_s^*\}\equiv\{\theta_s^*\left(\alpha^0\right)\}$. Then $\left(\{\theta_s\},\{\theta_s^*\}\right)$ is a Nash equilibrium that solves (56) and (57).*

Given the consumption sequence, $\{c\left(\alpha^0,y_t\right)\}$, and the sequence of foreign taxes, $\{\theta_s^*\}\equiv\{\theta_s^*\left(\alpha^0\right)\}$, let us construct the following sequence of prices:

$$p_t\equiv\beta^t\left[\prod_{s=0}^{t-1}\left(1-\theta_s^*\left(\alpha^0\right)\right)\right]\left[u^{*\prime}\left(Y-c\left(\alpha^0,y_t\right)\right)/u^{*\prime}\left(Y-c\left(\alpha^0,y_0\right)\right)\right]. \qquad (69)$$

By equation (68) in Step 3, we therefore have

$$\beta^t u'(c_t) = \mu u^{*\prime}(Y - c\left(\alpha^0, y_0\right)) p_t \left[1 - \frac{u^{*\prime\prime}(Y - c\left(\alpha^0, y_t\right))}{u^{*\prime}(Y - c\left(\alpha^0, y_t\right))} (c\left(\alpha^0, y_t\right) - y_t)\right].$$

By equation (66), we also know that

$$\prod_{s=0}^{t-1} \left(1 - \theta_s\left(\alpha^0\right)\right) = \frac{1 - \frac{u^{*\prime\prime}(Y - c\left(\alpha^0, y_0\right))}{u^{*\prime}(Y - c\left(\alpha^0, y_0\right))} (c\left(\alpha^0, y_0\right) - y_0)}{1 - \frac{u^{*\prime\prime}(Y - c\left(\alpha^0, y_t\right))}{u^{*\prime}(Y - c\left(\alpha^0, y_t\right))} (c\left(\alpha^0, y_t\right) - y_t)}.$$

This implies

$$\beta^t u'(c_t) = \lambda p_t \left[\prod_{s=0}^{t-1} \left(1 - \theta_s\left(\alpha^0\right)\right)\right]^{-1}, \tag{70}$$

with $\lambda \equiv \mu u^{*\prime}(Y - c\left(\alpha^0, y_0\right)) \left[u^{*\prime}(Y - c\left(\alpha^0, y_0\right)) - u^{*\prime\prime}(Y - c\left(\alpha^0, y_0\right))(c\left(\alpha^0, y_0\right) - y_0)\right] > 0$. Furthermore, by equations (64) and (69), we must have

$$\sum_{t=0}^{\infty} \beta^t p_t (c\left(\alpha^0, y_t\right) - y_t) = 0,$$

which further implies

$$\sum_{t=0}^{\infty} p_t \left[\prod_{s=0}^{t-1} \left(1 - \theta_s\left(\alpha^0\right)\right)\right]^{-1} (c\left(\alpha^0, y_t\right) - y_t) = L\left(\theta_s\left(\alpha^0\right), \theta_s^*\left(\alpha^0\right)\right). \tag{71}$$

since, by definition, $L\left(\theta_s\left(\alpha^0\right), \theta_s^*\left(\alpha^0\right)\right)$ are total tax revenues at Home. Thus the two necessary first-order conditions associated with (58) are satisfied. Since u is concave, these are sufficient as well. Thus $c \equiv \{c\left(\alpha^0, y_t\right)\}$ is a solution of (58) given prices $\{p_t\}$. The exact same argument implies that $c^* \equiv \{Y - c\left(\alpha^0, y_t\right)\}$ is a solution of of (59). By construction, we have $c_t + c_t^* = Y$ in all periods. Thus $p\left(\theta, \theta^*\right) \equiv \{p_t\}$ is a sequence of equilibrium prices. By definition, $(\{\theta_s\left(\alpha^0\right)\}, \{\theta_s^*\left(\alpha^0\right)\})$ is therefore a Nash equilibrium. ∎

Proof of Lemma 2. Lemma 2 directly follows from Step 1 in the proof of Lemma 1. ∎

Proof of Proposition 6. The foreign and domestic consumers' Euler equations imply

$$\frac{1 - \theta_t}{1 - \theta_t^*} = \frac{u'(c_t)}{u^{*\prime}(c_t^*)} \frac{u^{*\prime}(c_{t+1}^*)}{u'(c_{t+1})}.$$

Using the good market clearing condition (3), we can rearrange this expression as

$$\frac{1 - \theta_t}{1 - \theta_t^*} = \frac{u'(c_t)}{u^{*\prime}(Y - c_t)} \frac{u^{*\prime}(Y - c_{t+1})}{u'(c_{t+1})}.$$

By Lemma 2, we know that c_t is increasing in y_t. Since u and u^* are strictly concave, the previous expression therefore implies

$$\frac{1 - \theta_t}{1 - \theta_t^*} < 1 \text{ if and only if } y_t > y_{t+1}.$$

Proposition 6 directly derives from the previous equivalence. ∎